Overview of Calculation Approaches

Early Years into KS1

- Visualisation to secure understanding of the number system, especially the use of place value resources such as Base 10 and 100 Squares.
- Secure understanding of numbers to 10, using resources such as Tens Frames, fingers and multi-link.
- To begin making links between the different images of a number and their links to calculation.
- Practical, oral and mental activities to understand calculation.
- Personal methods of recording.

Key Stage 1

- Introduce signs and symbols ***(+ , -, x, ÷ in Year 1 and <, > signs in Year 2)***
- Extended visualisation to secure understanding of the number system beyond 100, especially the use of place value resources such as Base 10, Place Value Charts & Grids, Number Grids, Arrow Cards and Place Value Counters.
- Further work on recognising numbers without counting and Tens Frames to develop basic calculation understanding, supported by multi-link.
- Continued use of practical apparatus to support the early teaching of 2-digit calculation. For example, using base 10 to demonstrate partitioning and exchanging before these methods are taught as jottings / number sentences.
- Methods of recording / jottings to support calculation (e.g. partitioning or counting on).
- Use images such as empty number lines to support mental and informal calculation.

Year 3

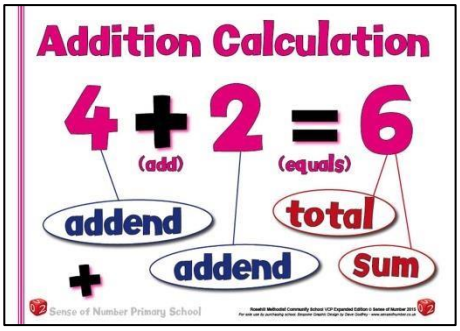
- Continued use of practical apparatus, especially Place Value Counters and Base 10 to visualise written / column methods before and as they are actually taught as procedures.
- Continued use of mental methods and jottings for 2 and 3 digit calculations.
- Introduction to more efficient informal written methods / jottings including expanded methods and efficient use of number lines (especially for subtraction).
- Column methods, where appropriate, for 3 digit additions and subtractions.

Years 3-6

- Continued use of mental methods for any appropriate calculation up to 6 digits.
- Standard written (compact) / column procedures to be learned for all four operations
- Efficient informal methods (expanded addition and subtraction, grid multiplication, division by chunking) and number lines are still used when appropriate. Develop these to larger numbers and decimals where appropriate.

N.B. Children must still be allowed access to practical resources to help visualise certain calculations, including those involving decimals

Addition Progression



The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Children need to acquire **one efficient written method of calculation** for addition that they know they can rely on **when mental methods are not appropriate.**

To add successfully, children need to be able to:

- **recall all addition pairs to 9 + 9 and complements in 10;**
- **add mentally a series of one-digit numbers, such as 5 + 8 + 4;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

Mental Addition Strategies

There are 5 key mental strategies for addition, which need to be a regular and consistent part of the approach to calculation in all classes from Year 2 upwards.

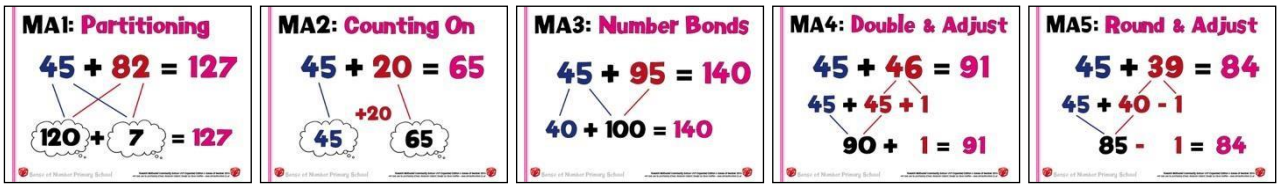
These strategies will be introduced individually when appropriate, and then be rehearsed and consolidated throughout the year until they are almost second nature.

These strategies are **partitioning, counting on, round and adjust, double and adjust and using number bonds.** The first two strategies are also part of the written calculation policy (see pages 12-14) but can equally be developed as simple mental calculation strategies once children are skilled in using them as jottings.

Using the acronym **RAPA CODA NUMBO**, children can be given weekly practice in choosing the most appropriate strategy whenever they are faced with a simple addition, usually of 2 or 3 digit numbers, but also spotting the opportunities (E.g. 3678 + 2997) when they can be used with larger numbers

RA	**Round & Adjust**
PA	**Partitioning**
CO	**Counting On**
DA	**Double & Adjust**
NUMBO	**Number Bonds**

For example, using the number 45, we can look at the other number chosen, and decide on the most appropriate mental calculation strategy.



The 5 key strategies need to be linked to the key messages from pages 2 and 3 –

The choice as to whether a child will choose to use a mental method or a jotting will depend upon

a) the numbers chosen and

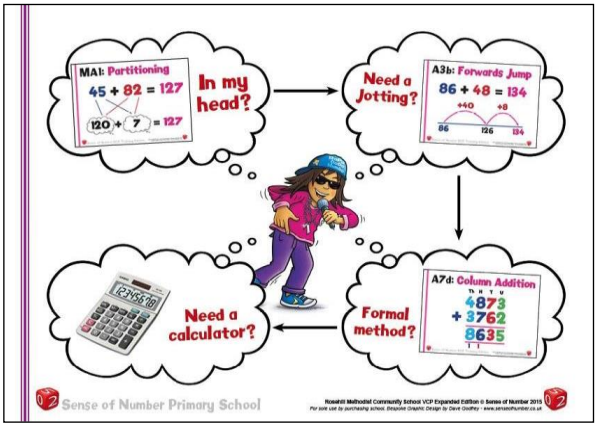
b) the level of maths that the child is working at.

For example, for **57 + 35**

a Year 2 child may use a long jotting or number line

a Year 3 child might jot down a quick partition jotting,

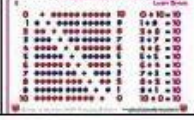
a Year 4 child could simply partition and add mentally.

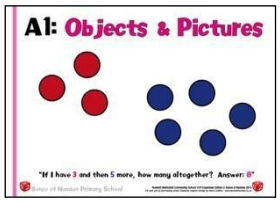
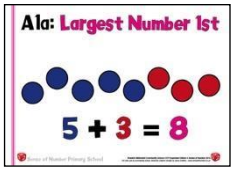
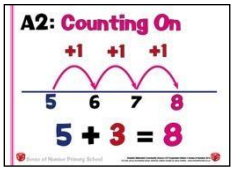
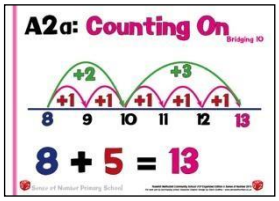
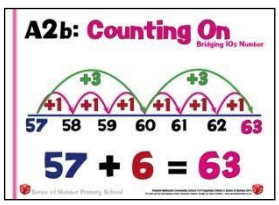


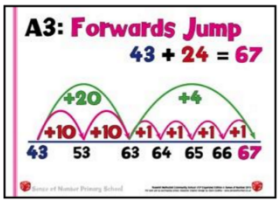
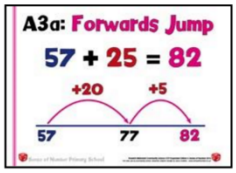
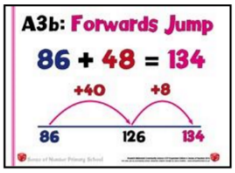
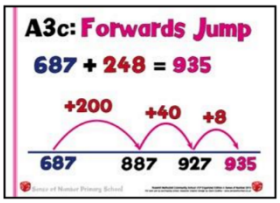
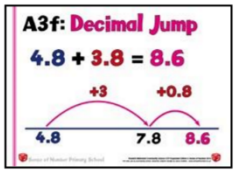
As a strategy develops, a child will begin to recognise the instances when it would be appropriate: -

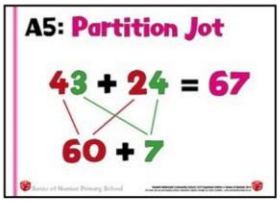
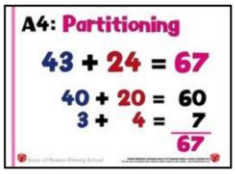
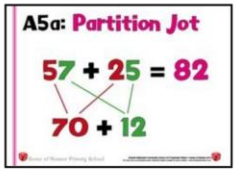
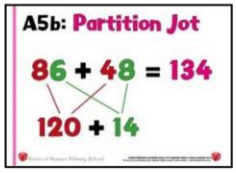
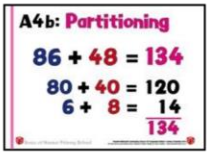
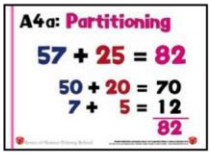
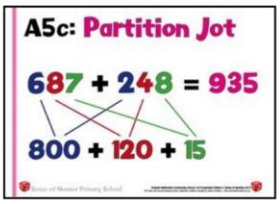
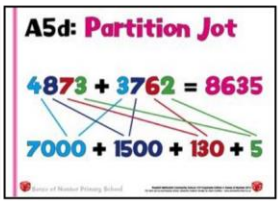
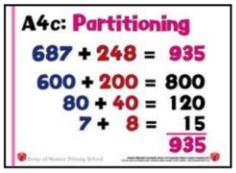
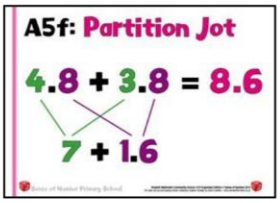
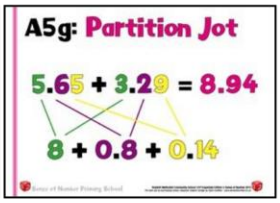
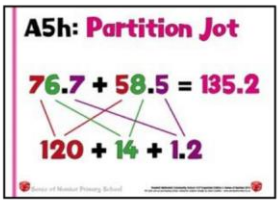
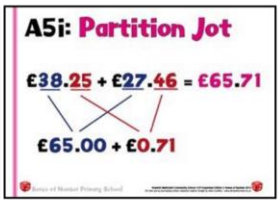
E.g. 27 +9, 434 + 197, 7.6 + 1.9 and 5.86 + 3.97 can all be calculated very quickly by using the **Round & Adjust** strategy.

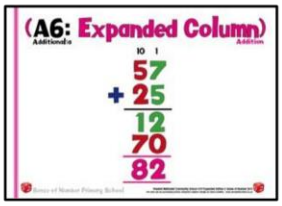
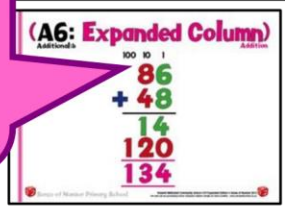
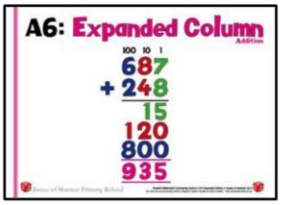
Below you can see the progression of each strategy through the year groups, with some appropriate examples of numbers, which may be used for each strategy.

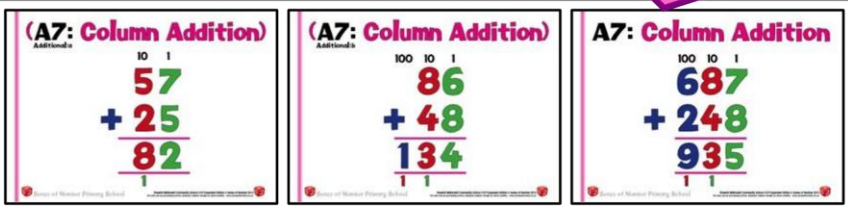
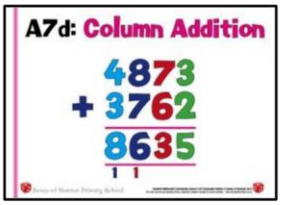
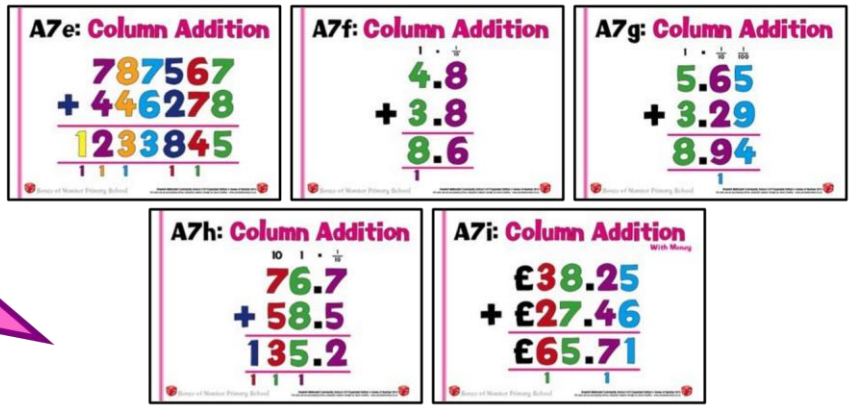
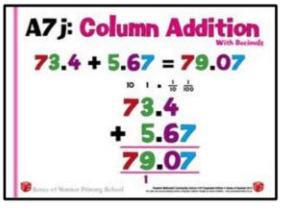
MA	**MA1: Partitioning** 45 + 82 = 127 120 + 7 = 127	**MA2: Counting On** 45 + 20 = 65 +20 45 65		**MA3: Number Bonds** 45 + 95 = 140 40 + 100 = 140	**MA4: Double & Adjust** 45 + 46 = 91 45 + 45 + 1 90 + 1 = 91	**MA5: Round & Adjust** 45 + 39 = 84 45 + 40 - 1 85 - 1 = 84
Y1		**MA2a: Counting On** 12 + 5 = 17 +5 12 17	**MA2b: Counting On** 57 + 10 = 67 +10 57 67	**MA3: Number Bonds** 	**MA4: Double & Adjust** 5 + 6 = 11 5 + 5 + 1 10 + 1 = 11	**MA5: Round & Adjust** 45 + 9 = 54 45 + 10 - 1 = 55 - 1 = 54
Y2	**MA1: Partitioning** 43 + 21 = 64 60 + 4 = 64	**MA2a: Counting On** 78 + 7 = 85 +7 78 85	**MA2b: Counting On** 58 + 40 = 98 +40 58 98	**MA3: Number Bonds** 3 + 4 + 7 = 14 10 4	**MA4: Double & Adjust** 7 + 8 = 15 7 + 7 + 1 14 + 1 = 15	**MA5: Round & Adjust** 45 + 19 = 64 45 + 20 - 1 65 - 1 = 64
Y3	**MA1: Partitioning** 57 + 25 = 82 70 + 12 = 82	**MA2a: Counting On** 85 + 50 = 135 +50 85 135	**MA2b: Counting On** 534 + 300 = 834 +300 534 834	**MA3: Number Bonds** 43 + 9 + 7 + 21 = 80 50 30	**MA4: Double & Adjust** 16 + 17 = 33 16 + 16 + 1 32 + 1 = 33	**MA5: Round & Adjust** 45 + 97 = 142 45 + 100 - 3 145 - 3 = 142
Y4	**MA1: Partitioning** 648 + 231 = 879 800 + 70 + 9 = 879	**MA2a: Counting On** 784 + 60 = 844 +60 784 844	**MA2b: Counting On** 4837 + 3000 = 8347 +3000 4837 7837	**MA3: Number Bonds** 42 + 16 + 28 + 54 = 140 70 70	**MA4: Double & Adjust** 37 + 38 = 75 37 + 37 + 1 74 + 1 = 75	**MA5: Round & Adjust** 345 + 298 = 643 345 + 300 - 2 645 - 2 = 643
Y5	**MA1: Partitioning** 576 + 258 = 834 700 + 120 + 14 = 834	**MA2a: Counting On** 837 + 500 = 1337 +500 837 1337	**MA2b: Counting On** 7583 + 5000 = 12583 +5000 7583 12583	**MA3: Number Bonds** £4.56 + £3.27 + £1.44 = £9.27 £6.00 £3.27	**MA4: Double & Adjust** 125 + 127 = 252 125 + 125 + 2 250 + 2 = 252	**MA5: Round & Adjust** 4645 + 1996 = 6641 4645 + 2000 - 4 6645 - 4 = 6641
Y6	**MA1: Partitioning** 4.73 + 2.21 = 6.94 6 + 0.9 + 0.04 = 6.94	**MA2a: Counting On** 43,826 + 30,000 = 73,826 +30,000 43,826 73,826	**MA2b: Counting On** 5,763,947 + 4,000,000 = 9,763,947 +4,000,000 5,763,947 9,763,947	**MA3: Number Bonds** 24.25 + 31.63 + 21.75 = 77.63 46 31.63	**MA4: Double & Adjust** 4.5 + 4.7 = 9.2 4.5 + 4.5 + 0.2 9 + 0.2 = 9.2	**MA5: Round & Adjust** 45.2 + 49.9 = 95.1 45.2 + 50 - 0.1 95.2 - 0.1 = 95.1

Stage 1	Finding a Total and the Empty Number Line	Alternative Method: Counting on Mentally or as a jotting
FS/Y1	Initially, children need to represent addition using a range of different resources, and understand that a total can be found by counting out the first number, counting out the second number then counting how many there are altogether. **3 + 5 = 8**	
		3 (held in head) then use fingers to count on 5 ("3… 4,5,6,7,8)
	This will quickly develop into placing the largest number first, either as a pictorial / visual method or by using a number line. **5 + 3 = 8**  	5 (held in head) then count on 3 ("5 … 6, 7, **8**")
Y1/2	Steps in addition can be recorded on a number line. The steps often bridge through 10. **8 + 5 = 13**	
		8 (held in head) then count on 5 ("8 … 9, 10, 11, 12, **13**")
	The next step is to bridge through a multiple of 10.	
		57 (held in head) then count on 6 ("57 … 58,59,60,61,62,63")
	The number line becomes a key image for demonstrating how to keep one number whole, whilst partitioning the other number. Teach the children firstly to add the tens then the ones individually (43 + 24 = 43 + 10 + 10 + 1 + 1 + 1 + 1) before progressing to counting on in tens and ones (43 + 20 + 4)	This method will be a jotting approach, and may look like the following examples: - 43 + 24 43 + 20 = 63 63 + 4 = 67

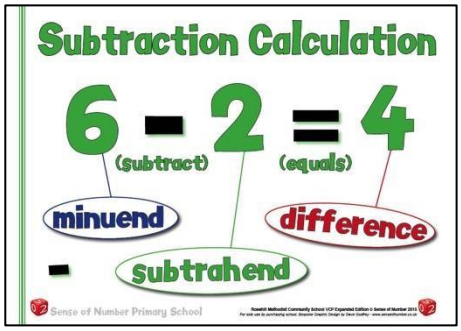
		Or 43 + 20 + 4 = 67
	Develop to crossing the 10s, then the 100s boundary **57 + 25 = 82** **86 + 48 = 134**	
	 	**57 + 25** 57 + 20 = 77 77 + 5 = 82 57 + 20 + 5 = 82 **86 + 48** 86 + 40 = 126 126 + 8 = 134 86 + 40 + 8 = 1
Y3/4	For some children, this method can still be used for 3 digit calculations	**Number lines support children's thinking if they find partitioning / column addition difficult, as it simply involves counting on in 100s, 10s & 1s.**
		**687 + 248** 687 + 200 = 887 887 + 40 = 927 927 + 8 = 935 Or 687 + 200 + 40 + 8 = 935
Y5/6	In Years 5 and 6, if necessary, children can return to this method to support their understanding of decimal calculation	
	 	**4.8 + 3.8** 4.8 + 3 = 7.8 7.8 + 0.8 = 8.6 Or 4.8 + 3 + 0.8 = 8.6 **Hopefully, with the above calculation, many children would mentally Round & Adjust (4.8 + 4 – 0.2 = 8.6)**

Stage 2	Partition Jot	Alternative Method: Traditional Partitioning
Y2/3	**Traditionally, partitioning has been presented using the method on the right. Although this does support place value and the use of arrow cards, it is very laborious, so it is suggested that adopting the 'partition jot' method will improve speed and consistency for mental to written (or written to mental) progression**	Record steps in addition using partition, initially as a jotting: - **43 + 24 =** 40 + 20 + 3 + 4 = 60 + 7 = 67 Or, preferably
	As soon as possible, refine this method to a much quicker and clearer '**Partition Jot**' approach 	
	As before, develop these methods, especially Partition Jot, towards crossing the 10s and then 100s.	
	 	 
	This method will soon become the recognised jotting to support the teaching of partitioning. It can be easily extended to 3 and even 4 digit numbers when appropriate.	For certain children, the traditional partitioning method can still be used for 3 digit numbers, but is probably too laborious for 4 digit numbers.
Y3/4	 	
	Partition jot is also extremely effective as a quicker alternative to column addition for decimals.	Some simple decimal calculations can also be completed this way.
Y5/6	 	
	For children with higher-level decimal place value skills, partition jot can be used with more complex decimal calculations or money.	
	 	

Stage 3	Expanded Method in Columns
Y3	**Column methods of addition are introduced in Year 3, but it is crucial that they still see mental calculation as their first principle, especially for 2 digit numbers.** **Column methods should only be used for more difficult calculations, usually with 3 digit numbers that cross the Thousands boundary or most calculations involving 4 digit numbers and above.** **N.B. Even when dealing with bigger numbers / decimals, children should still look for the opportunity to calculate mentally (E.g. 4675 + 1998)**
	2 digit examples are used below simply to introduce column methods to the children. Most children would continue to answer these calculations mentally or using a simple jotting.
	Using the column, children need to learn the principle of adding the ones first rather than the tens.
	The 'expanded' method is a very effective introduction to column addition. It continues to use the partitioning strategy that the children are already familiar with, but begins to set out calculations vertically. It is particularly helpful for automatically 'dealing' with the 'carry' digit
Y3/4	**A. Single 'carry' in units** / **B. 'Carry' in units and tens**
	 'Eighty plus forty equals one hundred and twenty, because 'eight plus four equals twelve. 
	Once this method is understood, it can quickly be adapted to using with three digit numbers. It is rarely used for 4 digits and beyond as it becomes too unwieldy.
	
	The time spent on practising the expanded method will depend on security of number facts recall and understanding of place value. Once the children have had enough experience in using expanded addition, and have also used practical resources (Base 10 / place value counters) to model exchanging in columns, they can be taken on to standard, 'traditional' column addition.

Stage 4	Column Method
Y3/4	As with the expanded method, begin with 2 digit numbers, simply to demonstrate the method, before moving to 3 digit numbers. **Make it <u>very clear</u> to the children that they are still expected to deal with all 2 digit (and many 3 digit) calculations mentally (or with a jotting),** and that the column method is designed for numbers that are too difficult to access using these ways. The column procedure **<u>is not</u>** intended for use with 2 digit numbers.
	'Carry' ones then ones and tens Use the words 'carry ten' and 'carry hundred', not 'carry one'
	Record carry digits below the line. 
Y4	Once confident, use with 4 digit numbers **(Year 4).**
	
Y5/6	Extend to 5/6 digit calculations then decimal calculations **(Year 5)**
	If children make repeated errors at any stage, they can return to the expanded method or an earlier jotting. 
	The key skill in upper Key Stage 2 that needs to be developed is the laying out of the column method for calculations with decimals in different places.
	

Subtraction Progression



The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

To subtract successfully, children need to be able to:

- **recall all addition and subtraction facts to 20;**
- **subtract multiples of 10 (such as 160 – 70) using the related subtraction fact (e.g. 16 – 7), and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Children need to acquire **one efficient written method of calculation for subtraction**, which they know they can rely on **when mental methods are not appropriate.**

NOTE: They should look at the actual numbers each time they see a calculation and decide whether or not their favoured method is most appropriate (e.g. If there are zeroes in a calculation such as 206 -198) then the 'counting on' approach may well be the best method in that particular instance).

Therefore, when subtracting, whether mental or written, children will mainly choose between two main strategies to find the difference between two numbers: -

Counting Back (Taking away) ***Counting On***

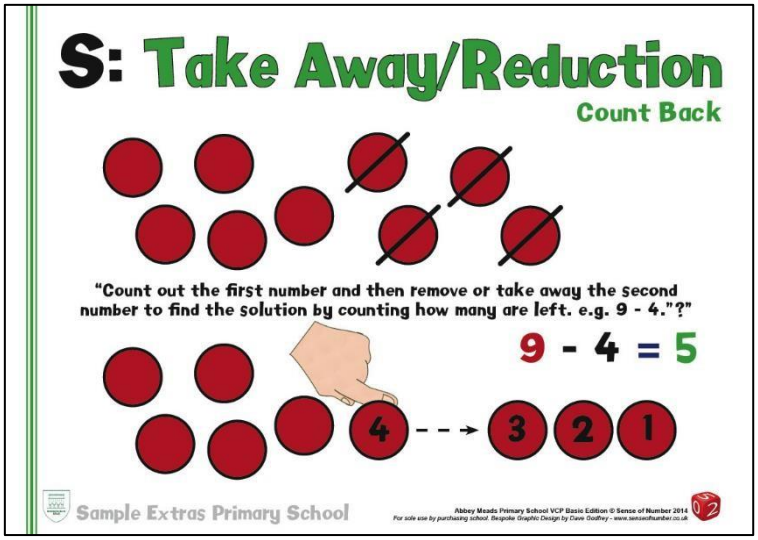
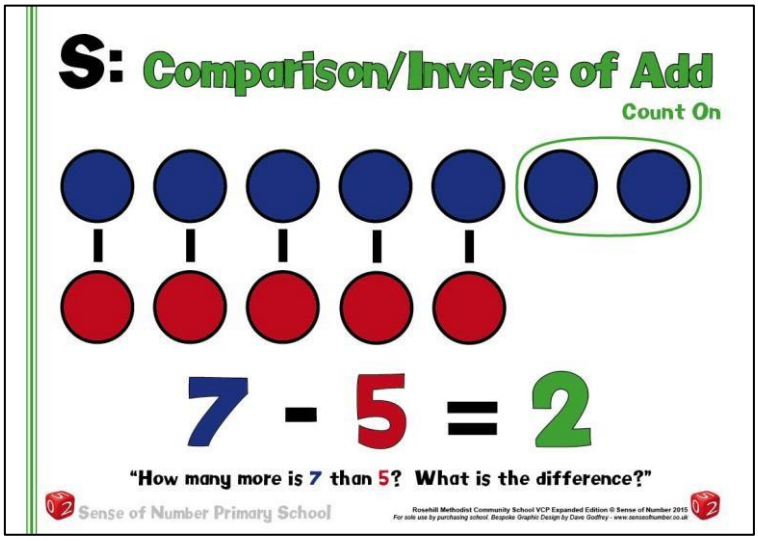
When should we count back and when should we count on?

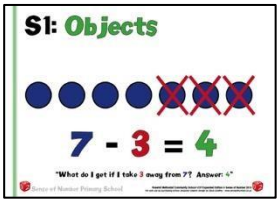
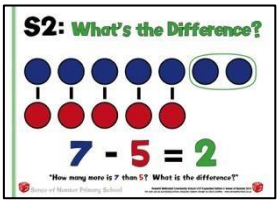
This will alter depending on the calculation (see below), but often the following rules apply;

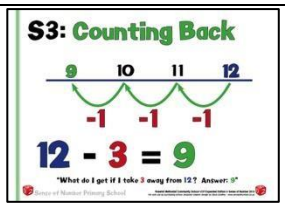
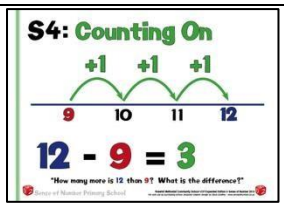
If the numbers are far apart, or there isn't much to subtract (278 – 24) then count back.

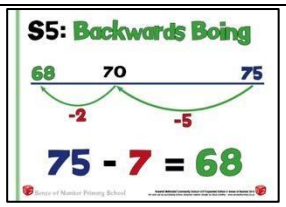
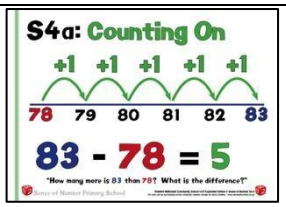
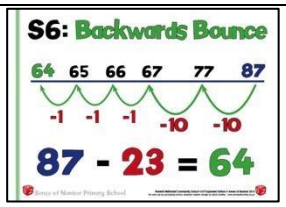
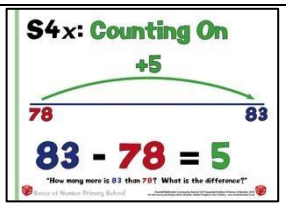
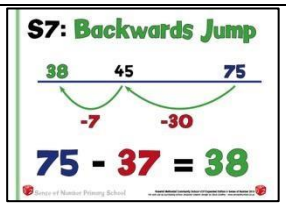
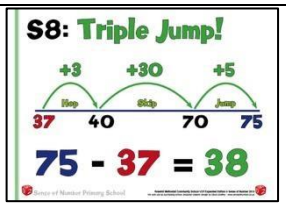
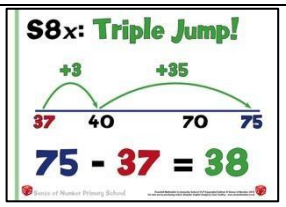
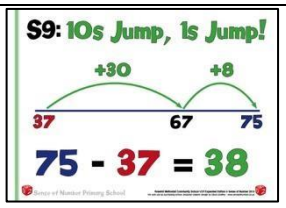
If the numbers are close together (206 – 188), then count up

In many cases, either strategy would be suitable, depending on preference (743 – 476)

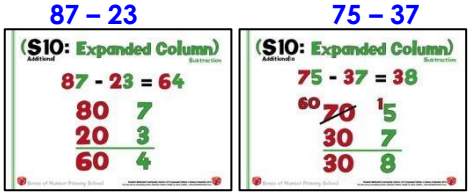
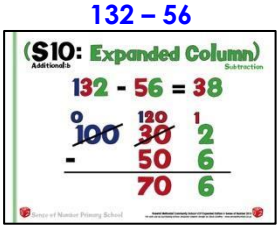
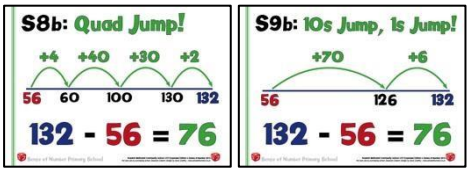
Models	Subtraction
Removing items from a set: **A: Take Away** **B: Reduction** (Count Back Images)	
	Take Away: Samir has 12 cakes and Nihal takes 5 cakes. How many cakes does Samir now have? **Reduction:** The shoes originally cost £12, but have been reduced in the sale by £5. How much do they now cost?
Comparing two sets: **A: Comparison** **B: Inverse of Addition** (Counting Up/On Images)	
	Comparision: Samir has 12 cakes and Nihal has 5 cakes. How many more cakes does Samir have than Nihal? **Inverse of Addition:** The shoes cost £12, but I've only got £5. How much more money will I need in order to buy the shoes? (5 + ? = 12)

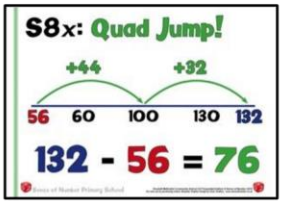
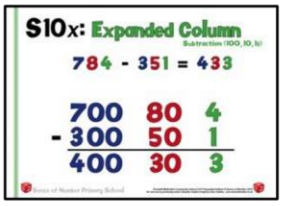
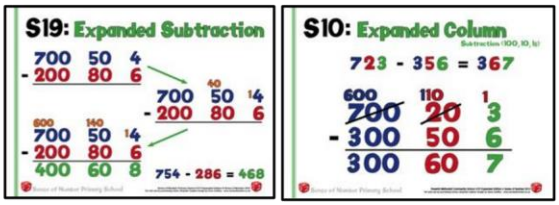
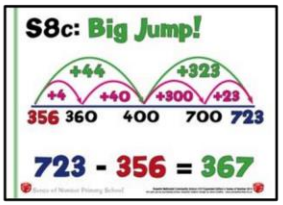
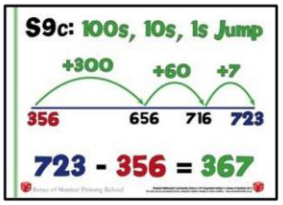
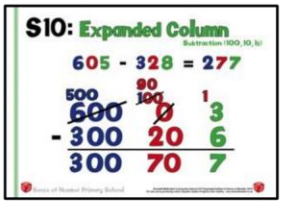
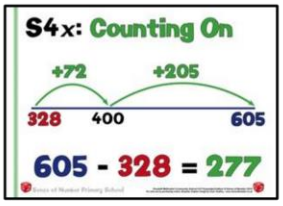
INTRO	Subtraction by counting back (or taking away)	Subtraction by counting up (or complementary addition)
FS/Y1	Early subtraction in EYFS will primarily be concerned with '***taking away***', and will be modelled using a wide range of models and resources.	
		
	This will continue in Year 1, using resources and images (including the desktop number track / line) to practise taking away practically, and then counting back on demarcated number lines.	In Year 1, it is also vital that children understand the concept of subtraction as ***'finding a difference'*** and realise that **<u>any</u>** subtraction can be answered in 2 different ways, either by counting up or counting back. Again, this needs to be modelled and consolidated regularly using a wide range of resources, especially multilink towers, counters and Numicon.
		

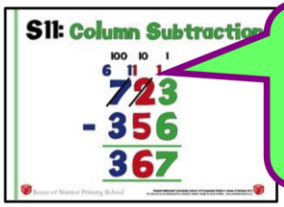
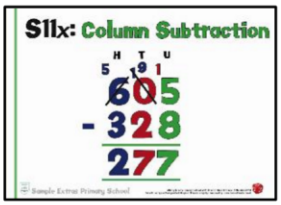
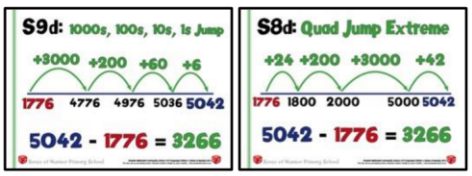
Stage 1	Using the empty number line	
	Subtraction by counting back (or taking away)	**Subtraction by counting up (or complementary addition)**
	The empty number line helps to record r explain the steps in mental subtraction. It is an ideal model for **counting back** an d **bridging ten**, as the steps can be shown cle arly. It can also show **counting up** from the smaller to the larger number to **find the diff rence.**	
Y1	The steps often bridge through a multiple of 10. **12 – 3 = 9**	Small differences can be found by counting up **12 – 9 = 3**
		

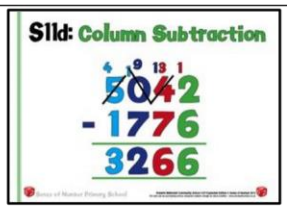
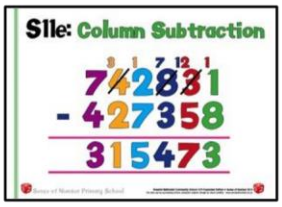
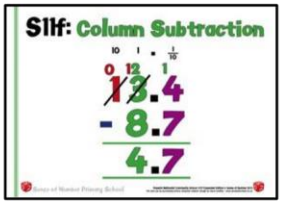
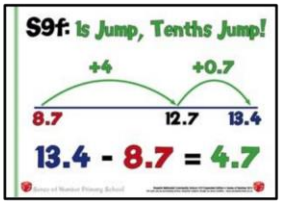
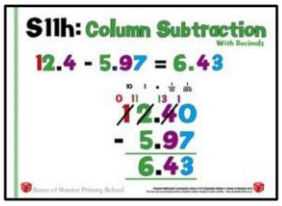
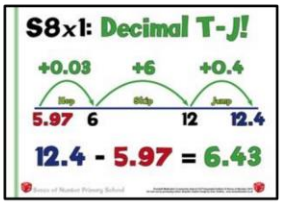
Y2/3	This is developed into crossing any multiple of 10 boundary. **75 – 7 = 68**	For 2 (or 3) digit numbers close together, count up **83 – 78 = 5** First, count in ones
		
	For 2 digit numbers, count back in 10s and 1s **87 – 23 = 64**	Then, use number facts to count in a single jump
		
	Then subtract tens and units in single jumps (87 – 20 – 3)	Continue to spot small differences with 3 digit numbers **(403 – 397 = 6)**
	Some numbers (75 – 37) can be subtracted just as quickly either way.	
	Either count back 30 then count back 7	Or count up from smaller to the larger number, initially with a 'triple jump' strategy of jumping to the next 10, then multiples of 10, then to the target number.
		
		This can also be done in 2 jumps.
		
		Some children prefer to jump in tens and ones, which is an equally valid strategy, as it links to the mental skill of *'counting up from any number in tens'*
		

Stage 2 — Expanded Method & Number Lines (continued)

	Subtraction by counting back Expanded Method	Subtraction by counting up Number Lines (continued)
	In Year 3, according to the New Curriculum, children are expected to be able to use both jottings <u>and</u> written column methods to deal with 3 digit subtractions. **This is only guidance, however – as long as children leave Year 6 able to access all four operations using formal methods, schools can make their own decisions as to when these are introduced.** **It is very important that they have had regular opportunities to use the number line 'counting up' approach first (right hand column below) so that they already have a secure method that is almost their first principle for most 2 and 3 digit subtractions.** This means that once they have been introduced to the column method they have an alternative approach that is often preferable depending upon the numbers involved. The number line method also gives those children who can't remember or successfully apply the column method an approach that will work with any numbers (even 4 digit numbers and decimals) if needed. It is advisable to spend at least the first two terms in Year 3 focusing upon the number line / counting up approach through regular practice, then introducing column method in the 3rd term as an alternative, or even waiting until Year 4 to introduce columns. **Ideally, whenever columns are introduced, the expanded method should be practised in depth (potentially up until 4 digit calculations are introduced)**	
Y3/4	**The expanded method of subtraction is an excellent way to introduce the column approach as it maintains the place value and is much easier to model practically with place value equipment such as Base 10 or place value counters** Introduce the expanded method with 2 digit numbers, but only to explain the process. Column methods are very rarely needed for 2 digit calculations. Partition both numbers into tens and ones, firstly with no exchange then exchanging from tens to the ones. 87 – 23 75 – 37 	
	Develop into exchanging from hundreds to tens and tens to ones. 132 – 56 	The number line method is equally as effective when crossing the hundreds boundary, either by the triple / quad jump strategy or by counting in tens then ones. 

		The 'quad jump' can be completed by many children in fewer steps, either a triple or **double** jump. 
A	Take the method into three digit numbers. Subtract the ones, then the tens, then the hundreds. Demonstrate without exchanging first. **784 – 351**	
		***For examples without exchanging, the number line method takes considerably longer than mental partitioning or expanded.***
B	Move towards exchanging from hundreds to tens and tens to ones, in two stages if necessary **723 – 356**	The example below shows 2 alternatives, for children who need different levels of support from the image.
		
		As before, many children prefer to count in hundreds, then tens, then ones.
	For examples where exchanging is needed, then the number line method is equally as efficient, and is often easier to complete	
C	Use some examples which include the use of zeros e.g. **605 – 328**.	***For numbers containing zeros, counting up is often the most reliable method.***
		
	Continue to use expanded subtraction until both number facts and place value are considered to be very secure!	

Stage 3	Standard Column Method (decomposition)	
	Subtraction by counting back Standard Method	**Subtraction by counting up Number Lines (continued)**
Mainly **Y4+**	**Decomposition relies on secure understanding of the expanded method, and simply displays the same numbers in a contracted form.** As with expanded method, and using practical resources such as place value counters to support the teaching, children in Years 3 or 4 (depending when the school introduces the column procedure) will quickly move from decomposition via 2-digit number 'starter' examples to 2 / 3 digit and then 3 digit columns. **75 – 37** **132 – 56** 	
	723 – 356  ***Continue to refer to digits by their actual value, not their digit value, when explaining a calculation. E.g. One hundred and twenty subtract fifty.***	
	Again, use examples containing zeros, remembering that it may be easier to count on with these numbers (see Stage 2) **605 – 328** 	**It is even possible, for children who find column method very difficult to remember, or who regularly make the same mistakes, to use the number line method for 4 digit numbers, using either of the approaches.**
Y4	From late Y4 onwards, move onto examples using 4 digit (or larger) numbers and then onto decimal calculations. **5042 – 1776**	**5042 – 1776** 

		
Y5/6	In Years 5 & 6 apply to any 'big number' examples.	
		
	Both methods can be used with decimals, although the counting up method becomes less efficient and reliable when calculating with more than two decimal places.	
	13.4 – 8.7	13.4 – 8.7
		
	12.4 – 5.97	12.4 – 5.97
		
	72.43 – 47.85