



## Overview of Calculation Approaches

### Early Years into KS1

- Visualisation to secure understanding of the number system, especially the use of place value resources such as Base 10 and 100 Squares.
- Secure understanding of numbers to 10, using resources such as Tens Frames, fingers and multi-link.
- To begin making links between the different images of a number and their links to calculation.
- Practical, oral and mental activities to understand calculation.
- Personal methods of recording.

### Key Stage 1

- Introduce signs and symbols ( **$+$ ,  $-$ ,  $\times$ ,  $\div$  in Year 1 and  $<$ ,  $>$  signs in Year 2**)
- Extended visualisation to secure understanding of the number system beyond 100, especially the use of place value resources such as Base 10, Place Value Charts & Grids, Number Grids, Arrow Cards and Place Value Counters.
- Further work on recognising numbers without counting and Tens Frames to develop basic calculation understanding, supported by multi-link.
- Continued use of practical apparatus to support the early teaching of 2-digit calculation. For example, using base 10 to demonstrate partitioning and exchanging before these methods are taught as jottings / number sentences.
- Methods of recording / jottings to support calculation (e.g. partitioning or counting on).
- Use images such as empty number lines to support mental and informal calculation.

### Year 3

- Continued use of practical apparatus, especially Place Value Counters and Base 10 to visualise written / column methods before and as they are actually taught as procedures.
- Continued use of mental methods and jottings for 2 and 3 digit calculations.
- Introduction to more efficient informal written methods / jottings including expanded methods and efficient use of number lines (especially for subtraction).
- Column methods, where appropriate, for 3 digit additions and subtractions.

### Years 3-6

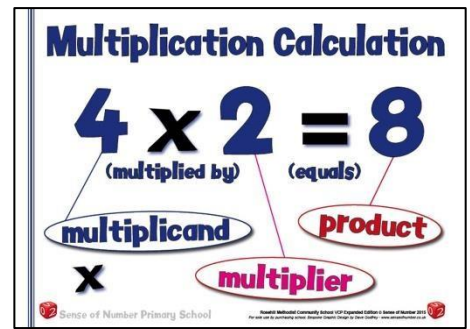
- Continued use of mental methods for any appropriate calculation up to 6 digits.
- Standard written (compact) / column procedures to be learned for all four operations
- Efficient informal methods (expanded addition and subtraction, grid multiplication, division by chunking) and number lines are still used when appropriate. Develop these to larger numbers and decimals where appropriate.

N.B. Children must still be allowed access to practical resources to help visualise certain calculations, including those involving decimals



# Multiplication Progression

*The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.*



These notes show the stages in building up to using an efficient method for

- two-digit by one-digit multiplication by the end of Year 3,
- three-digit by one-digit multiplication by the end of Year 4,
- four-digit by one-digit multiplication **and** two/three-digit by two-digit multiplication by the end of Year 5
- three/four-digit by two-digit multiplication **and** multiplying 1-digit numbers with up to 2 decimal places by whole numbers by the end of Year 6.

To multiply successfully, children need to be able to:

- recall all multiplication facts to  $12 \times 12$ ;
- partition numbers into multiples of one hundred, ten and one;
- work out products such as  $70 \times 5$ ,  $70 \times 50$ ,  $700 \times 5$  or  $700 \times 50$  using the related fact  $7 \times 5$  and their knowledge of place value;
- similarly apply their knowledge to simple decimal multiplications such as  $0.7 \times 5$ ,  $0.7 \times 0.5$ ,  $7 \times 0.05$ ,  $0.7 \times 50$  using the related fact  $7 \times 5$  and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as  $60 + 70$ ) or of 100 (such as  $600 + 700$ ) using the related addition fact,  $6 + 7$ , and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

## Note:

Children need to acquire **one efficient written method of calculation for multiplication, which** they know they can rely on **when mental methods are not appropriate.**

**It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.**

These mental methods are often more efficient than written methods when multiplying.




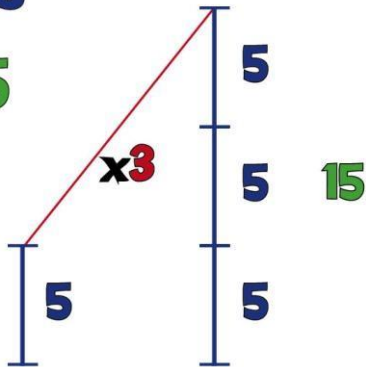


*Use partitioning and grid methods until number facts and place value are secure*

*For a calculation such as  $25 \times 24$ , a quicker method would be 'there are four 25s in 100 so  $25 \times 24 = 100 \times 6 = 600$ '*

*When multiplying a 3 / 4 digit x 2-digit number the standard method is usually the most efficient*

*At all stages, use known facts to find other facts.  
E.g. Find  $7 \times 8$  by using  $5 \times 8$  (40) and  $2 \times 8$  (16)*



Models	Multiplication
<h1>Repeated Addition</h1>	<div data-bbox="778 241 1460 347"> <h2>M: Repeated Addition</h2> <p>(Groups)</p> </div> <div data-bbox="783 389 1449 573">  </div> <div data-bbox="778 618 1460 683"> <math display="block">5 \times 3 = 5 + 5 + 5 = 15</math> </div> <div data-bbox="842 689 1396 719"> <p>"5 multiplied by 3" means "5, 3 times", which gives "3 lots of 5!"</p> </div> <div data-bbox="766 712 1460 748"> <p> Sense of Number Primary School <small>Rosehill Methodist Community School VCP Expanded Edition © Sense of Number 2015. For sale use by purchasing school. Design: Graphics Design by Dave Giffney - www.senseofnumber.co.uk</small> </p> </div>
<h1>Scaling</h1>	<div data-bbox="778 846 1093 922"> <h2>M: Scaling</h2> </div> <div data-bbox="778 963 1077 1025"> <math display="block">5 \times 3 = 15</math> </div> <div data-bbox="1070 913 1437 1279">  </div> <div data-bbox="863 1294 1380 1326"> <p>"5 multiplied by 3" means "5, 3 times as big!"</p> </div> <div data-bbox="766 1317 1460 1352"> <p> Sense of Number Primary School <small>Rosehill Methodist Community School VCP Expanded Edition © Sense of Number 2015. For sale use by purchasing school. Design: Graphics Design by Dave Giffney - www.senseofnumber.co.uk</small> </p> </div>

# Mental Multiplication





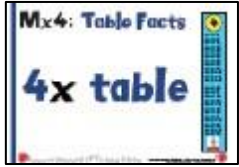
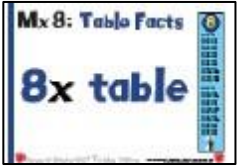
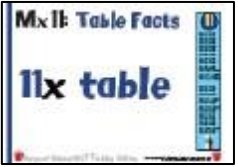



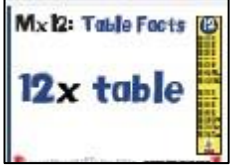
In a similar way to addition, multiplication has a range of mental strategies that need to be developed both before and then alongside written methods (both informal and formal).

## Tables Facts

In Key Stage 2, however, before any written methods can be securely understood, children need to have a bank of multiplication tables facts at their disposal, which can be recalled instantly.

The learning of tables facts does begin with counting up in different steps, but by the end of Year 4 it is expected that most children can instantly recall all facts up to  $12 \times 12$ .

The progression in facts is as follows (11's moved into Y3 as it is a much easier table to recall): -

<b>Y2</b>				
<b>Y3</b>				
<b>Y4</b>				

Once the children have established a bank of facts, they are ready to be introduced to jottings and eventually written methods.

## Doubles & Halves

The other facts that children need to know by heart are doubles and halves. These are no longer mentioned explicitly within the National Curriculum, making it even more crucial that they are part of a school's mental calculation policy. If children haven't learned to recall simple doubles instantly, and haven't been taught strategies for mental doubling, then they cannot access many of the mental calculation strategies for multiplication (E.g. Double the 4 times table to get the 8 times table. Double again for the 16 times table etc.).

As a general guidance, children should know the following doubles: -

Year Group	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Doubles and Halves	All doubles and halves from double 1 to double 10 / half of 2 to half of 20	All doubles and halves from double 1 to double 20 / half of 2 to half of 40 (E.g. double 17=34, half of 28 = 14)	Doubles of all numbers to 100 with units digits 5 or less, and corresponding halves (E.g. Double 43, double 72, half of 46)  Reinforce doubles & halves of all multiples of 10 & 100 (E.g. double 800, half of 140)	Addition doubles of numbers 1 to 100 (E.g. 38 + 38, 76 + 76) and their corresponding halves  Revise doubles of multiples of 10 and 100 and corresponding halves	Doubles and halves of decimals to 10 – 1 d.p. (E.g. double 3.4, half of 5.6)	Doubles and halves of decimals to 100 – 2 d.p. (E.g. double 18.45, half of 6.48)

Before certain doubles / halves can be recalled, children can use a simple jotting to help them record their steps towards working out a double / half

**Y2**

**MM5: Doubling**  
Double **17** = **34**  
 $20 + 14 = 34$

**MM5a: Doubling**  
Double **37** = **74**  
 $60 + 14 = 74$

**Y3**

**MM5b: Doubling**  
Double **78** = **156**  
 $140 + 16 = 156$

**MM5c: Doubling**  
Double **340** = **680**  
 $600 + 80 = 680$

**MM5d: Doubling**  
Double **480** = **960**  
 $800 + 160 = 960$

**Y4**

**MM5e: Doubling**  
Double **278** = **556**  
 $400 + 140 + 16 = 556$

**Y4/5**

**MM5f: Doubling**  
Double **768** = **1536**  
 $1400 + 120 + 16 = 1536$

**MM5g: Doubling**  
Double **3.7** = **7.4**  
 $6 + 1.4 = 7.4$

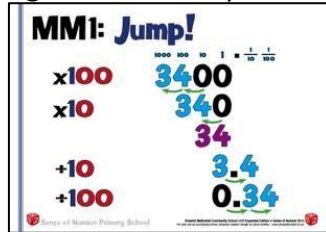
As mentioned, though, there are also several mental calculation strategies that need to be taught so that children can continue to begin any calculation with the question 'Can I do it in my head?' The majority of these strategies are usually taught in Years 4 – 6,

but there is no reason why some of them cannot be taught earlier as part of the basic rules of mathematics.

### Multiplying by 10 / 100 / 1000

The first strategy is usually part of the Year 5 & 6 teaching programme for decimals, namely that digits move to the left when multiplying by 10, 100 or 1000, and to the right when dividing.

This also secures place value by emphasising that the decimal point doesn't ever move, and that the digits move around the decimal point (not the other way round, as so many adults were taught at school).

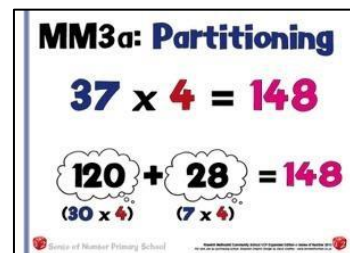
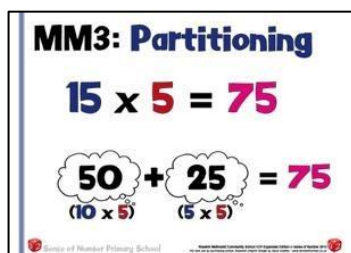


It would be equally beneficial to teach a simplified version of this strategy in KS1 / Lower KS2, encouraging children to move digits into a new column, rather than simply 'adding zeroes' when multiplying by 10/100.

The following 3 strategies can be explicitly linked to 3 of the strategies in mental addition (**Partitioning**, **Round & Adjust** and **Number Bonds**)

**Partitioning** is an equally valuable strategy for multiplication, and can be quickly developed from a jotting to a method completed entirely mentally. It is clearly linked to the grid method of multiplication, but should also be taught as a 'partition jot' so that children, by the end of Year 4, have become skilled in mentally partitioning 2 and 3 digit numbers when multiplying (with jottings when needed).

By the time they leave Year 6 they should be able to mentally partition most simple 2 & 3 digit, and also decimal multiplications.





**Round & Adjust** is also a high quality mental strategy for multiplication, especially when dealing with money problems in upper KS2. Once children are totally secure with rounding and adjusting in addition, they can be shown how the strategy extends into multiplication, where they round then adjust by the multiplier.

E.g. For  $39 \times 6$  round to  $40 \times 6$  (240) then adjust by  $1 \times 6$  (6) to give a product of  $240 - 6 = 234$ .

**MM4: Round & Adjust**

$$49 \times 3 = 147$$

$$(50 \times 3) - (1 \times 3)$$

$$150 - 3 = 147$$

**MM4a: Round & Adjust**

$$198 \times 4 = 792$$

$$(200 \times 4) - (2 \times 4)$$

$$800 - 8 = 792$$

**MM4b: Round & Adjust**

$$3.9 \times 5 = 19.5$$

$$(4 \times 5) - (0.1 \times 5)$$

$$20 - 0.5 = 19.5$$

**MM4c: Round & Adjust**

$$£5.99 \times 6 = £35.94$$

$$(£6 \times 6) - (1p \times 6)$$

$$£36 - 6p = £35.94$$

**Y4**

**Y4/5**

**Y5**

**Y5/6**

**Re-ordering** is similar to **Number Bonds** in that the numbers are calculated in a different order. I.e. The children look at the numbers that need to be multiplied, and, using commutativity, rearrange them so that the calculation is easier.

The asterisked calculation in each of the examples below is probably the easiest / most efficient rearrangement of the numbers.

**MM2: Re-ordering**

$$(9 \times 2) \times 5$$

$$18 \times 5 = 90$$

$$(9 \times 5) \times 2$$

$$45 \times 2 = 90$$

$$(2 \times 5) \times 9$$

$$10 \times 9 = 90 *$$

**MM2a: Re-ordering**

$$(7 \times 4) \times 5$$

$$28 \times 5 = 140$$

$$(7 \times 5) \times 4$$

$$35 \times 4 = 140$$

$$(4 \times 5) \times 7$$

$$20 \times 7 = 140 *$$

**MM2b: Re-ordering**

$$(9 \times 8) \times 6$$

$$72 \times 6 = 432$$

$$(9 \times 6) \times 8$$

$$54 \times 8 = 432 *$$

$$(8 \times 6) \times 9$$

$$48 \times 9 = 432$$

**Doubling**

strategies are probably the most crucial of the mental strategies for multiplication, as they can make difficult long multiplication calculations considerably simpler.

Initially, children are taught to double one table to find another (E.g..doubling the 3s to get the 6s) This can then be applied to any table: -

**MM6: Doubling Table Facts**

$$16 \times 7 = 112$$

$$(8 \times 2)$$

$$8 \times 7 = 56$$

$$\downarrow \quad \quad \downarrow \times 2$$

$$16 \times 7 = 112$$

**Doubling Up** enables multiples of 4, 8 and 16 onwards to be calculated by constant doubling: -

**MM7: Doubling Up**

$$17 \times 4 = 68$$

$$\text{Double } 17 = 34 \quad (17 \times 2)$$

$$\text{Double } 34 = 68 \quad (17 \times 4)$$

**MM7a: Doubling Up**

$$36 \times 8 = 112$$

$$\text{Double } 36 = 72 \quad (36 \times 2)$$

$$\text{Double } 72 = 144 \quad (36 \times 4)$$

$$\text{Double } 144 = 288 \quad (36 \times 8)$$

**MM7b: Doubling Up**

$$125 \times 16 = 2000$$

$$\text{Double } 125 = 250 \quad (125 \times 2)$$

$$\text{Double } 250 = 500 \quad (125 \times 4)$$

$$\text{Double } 500 = 1000 \quad (125 \times 8)$$

$$\text{Double } 1000 = 2000 \quad (125 \times 16)$$



**Doubling & Halving** is probably the best strategy available for simplifying a calculation. Follow the general rule that if you double one number within a multiplication, and halve the other number, then the product stays the same.

**MM9: Doubling & Halving**

$$\begin{array}{l} 45 \times 14 \\ 90 \times 7 = 630 \end{array}$$

**MM9a: Doubling & Halving**

$$\begin{array}{l} 36 \times 25 \\ 18 \times 50 \\ 9 \times 100 = 900 \end{array}$$

**MM9b: Doubling & Halving**

$$\begin{array}{l} 26 \times 32 \\ 52 \times 16 \\ 104 \times 8 = 832 \\ 208 \times 4 \text{ etc.} \end{array}$$

**Multiplying by 10 / 100 / 1000 then halving.** The final doubling / halving strategy works on the principle that multiplying by 10 / 100 is straightforward, and this can enable you to easily multiply by 5, 50 or 25.

**MM8: Mult by 10 then Halve**

$$\begin{array}{l} 86 \times 5 = 430 \\ 86 \times 10 = 860 \\ 860 \div 2 = 430 \end{array}$$

**MM8a: Mult by 100 then Halve**

$$\begin{array}{l} 56 \times 25 = 1400 \\ 56 \times 100 = 5600 \\ 5600 \div 2 = 2800 \\ 2800 \div 2 = 1400 \end{array}$$

**Factorising** The only remaining mental strategy, which again can simplify a calculation, is factorising. Multiplying a 2-digit number by 36, for example, may be easier if multiplying by a factor pair of 36 (x6 then x6, or x9 then x4, even x12 then x3)

**MM10: Factorising**

$$\begin{array}{l} 32 \times 15 = 480 \\ (32 \times 5 \times 3) \\ 160 \times 3 = 480 \end{array}$$

**MM10a: Factorising**

$$\begin{array}{l} 52 \times 24 = 1248 \\ (52 \times 4 \times 6) \\ 208 \times 6 = 1248 \end{array}$$

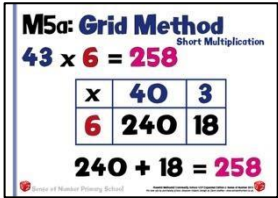
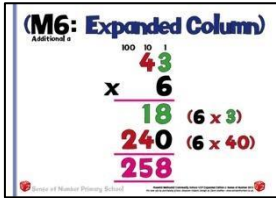
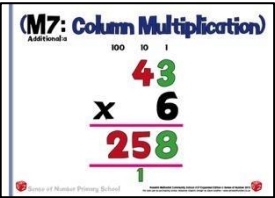
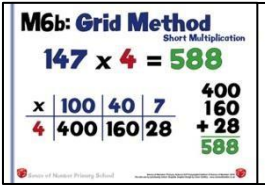
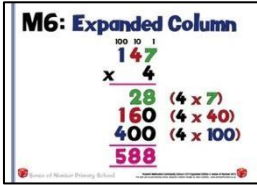
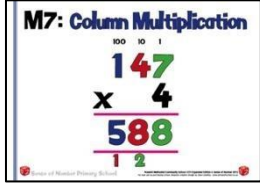
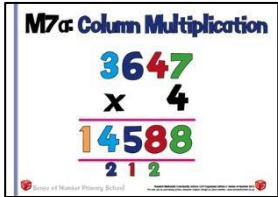
# Written Multiplication

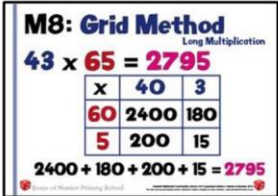
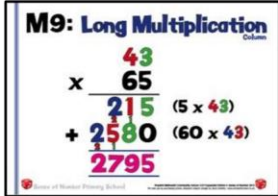
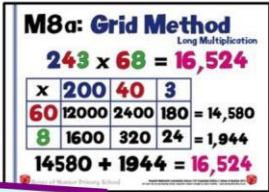
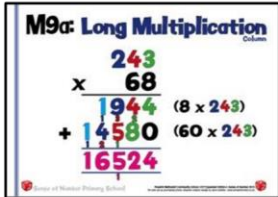
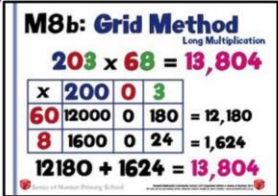
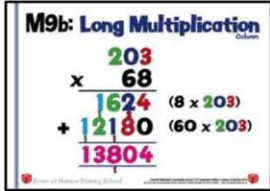
Stage 1	Number Lines, Arrays & Mental Methods
<b>FS</b>	<p>In Early Years, children are introduced to grouping, and are given regular opportunities to put objects into groups of 2, 3, 4, 5 and 10. They also stand in different sized groups, and use the term 'pairs' to represent groups of 2.</p> <p>Using resources such as Base 10 apparatus, Numicon, multi-link or an abacus, children visualise counting in ones, twos, fives and tens, saying the multiples as they count the pieces. E.g. Saying '10, 20, 30' or 'Ten, 2 tens, 3 tens' whilst counting Base 10 pieces</p>
<b>Y1</b>	<p>Begin by introducing the concept of multiplication as repeated addition.</p> <p>Children will make and draw objects in groups (again using resources such as Numicon, counters and multi-link), giving the product by counting up in 2s, 5s, 10s and beyond, and writing the</p> <div data-bbox="810 607 1083 799" data-label="Image"> </div> <p>Extend into making multiplication statements for 3s and 4s, using resources (especially real life equipment such as cups, cakes, sweets etc.)</p> <p><b>Make sure from the start that all children say the multiplication fact the correct way round, using the word 'multiply' more often than the word 'times'.</b></p> <p>For the example above, there are 5 counters in 2 groups, showing <b>5 multiplied by 2 (5x2)</b>, not 2 times 5. It is the '<b>5</b>' which is being scaled up / made bigger / <b>multiplied</b>.</p> <p><b>'5 multiplied by 2' shows '2 groups of 5' or 'Two fives'</b></p>
	<p>Develop the use of the array to show linked facts (commutativity).</p> <p>Emphasise that all multiplications can be worked out either way. (<math>2 \times 5 = 5 \times 2 = 10</math>)</p> <div data-bbox="778 1234 1051 1426" data-label="Image"> </div>
<b>Y2</b>	<p>Build on children's understanding that multiplication is repeated addition, using arrays and number lines to support the thinking. Explore arrays in real life.</p> <div data-bbox="778 1518 1051 1711" data-label="Image"> </div> <p>Start to emphasise commutativity, e.g. that <math>5 \times 3 = 3 \times 5</math></p>
<p><b>Continue to emphasise multiplication the correct way round.</b>  <b>E.g. <math>5 \times 3 = 5 + 5 + 5</math></b>  <b>5 multiplied by 3 = 15</b></p>	<div data-bbox="536 1818 809 2011" data-label="Image"> </div> <div data-bbox="853 1818 1126 2011" data-label="Image"> </div> <div data-bbox="1169 1818 1442 2011" data-label="Image"> </div>



Stage 2	Written Methods - Short Multiplication	
	Grid Multiplication (Mental 'Jotting')	Column multiplication (Expanded method into standard)
	<p>The grid method of multiplication is a simple, alternative way of recording the jottings shown previously.</p> <p><i>If necessary (for some children) it can initially be taught using an array to show the actual product.</i></p>	<p>The expanded method links the grid method to the standard method.</p> <p>It still relies on partitioning the tens and units, but sets out the products vertically.</p> <p>Children will use the expanded method until they can securely use and explain the standard method.</p>
Y3	<p>It is recommended that the grid method is used as the main method within Year 3. It clearly maintains place value, and helps children to visualise and understand the calculation better.</p>	<p>At some point within the year (preferably the 3<sup>rd</sup> term), the column method can be introduced, and children given the choice of using either grid or standard. Some schools may delay the introduction of column method until Year 4</p>
		<p>When setting out calculations vertically, begin with the ones first (as with addition and subtraction).</p>
Y4	<p>Continue to use both grid and column methods in Year 4 for more difficult 2 digit x 1 digit calculations, extending the use of the grid method into mental partitioning for those children who can use the method this way.</p> <p>At this point, the expanded method can still be used when necessary (to help 'bridge' grid with column), but children should be encouraged to use their favoured method (grid or column) whenever possible.</p>	

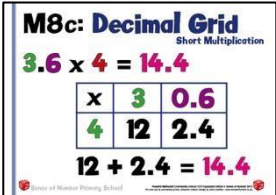
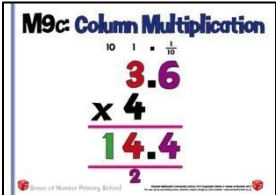
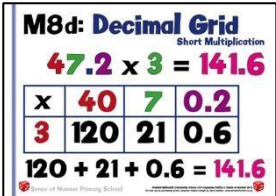
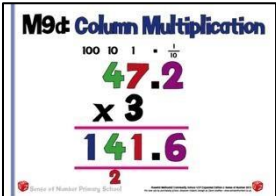
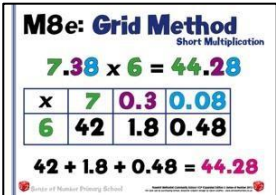
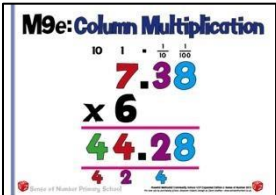
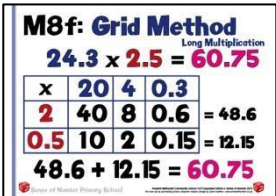
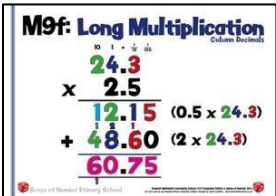
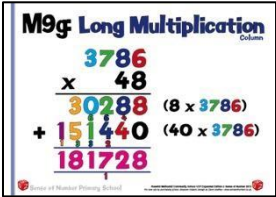


		 
	<p>For 3 digit x 1 digit calculations, both grid and standard methods are efficient. Continue to use the grid method to aid place value and mental arithmetic. Develop column method for speed, and to make the transition to long multiplication easier.</p> <p>If both methods are taught consistently then children in Year 4 will have a clear choice of 2 secure methods, and will be able to develop both accuracy and speed in multiplication.</p> <div style="border: 2px solid purple; border-radius: 15px; padding: 10px; width: fit-content; margin: 10px auto;"> <p><i>If children find it difficult to add the 'part products' then set them out vertically (or encourage column method)</i></p> </div>	
		 
<b>Y5</b>	<p>For a 4 digit x 1 digit calculation, the column method, once mastered, is quicker and less prone to error. The grid method may continue to be the main method used by children who find it difficult to remember the column procedure, or children who need the visual link to place value.</p> <div style="text-align: right;">  </div>	

Stage 3	Long Multiplication (TO x TO)	
	Grid Multiplication	Column multiplication (Expanded method into standard)
<b>Y5</b>	<p>Extend the grid method to TO x TO, asking children to estimate first so that they have a general idea of the answer.</p> <p><i>(43 x 65 is approximately 40 x 70 = 2800.)</i></p>  <p>As mentioned earlier, the grid method is often the 'choice' of many children in Years 5 and 6, due to its ease in both procedure and understanding / place value and is the method that they will mainly use for simple long multiplication calculations.</p>	<p>Children should only use the 'standard' column method of long multiplication if they can regularly get the correct answer using this method.</p>  <p>There is no 'rule' regarding the position of the 'carry' digits. Each choice has advantages and complications.</p> <p>Either carry the digits mentally or have your own favoured position for these digits.</p>
<b>Y6</b>	<p>For 3 digit x 2 digit calculations, grid method is quite inefficient, and has much scope for error due to the number of 'part-products' that need to be added.</p> <p><b>Use this method when you find the standard method to be unreliable or difficult to remember.</b></p>	<p><b>Most children, at this point, should be encouraged to choose the standard method.</b> For 3 digit x 2 digit calculations it is especially efficient, and less prone to errors when mastered.</p> <p><b>Although they may find the grid method easier to apply, it is much slower / less efficient.</b></p>
		
<p>Add these numbers for the overall product</p>		
	<p>Many children will find the use of Grid method as an efficient method for multiplying decimals.</p>	<p>Extend the use of standard method into the use of decimals.</p>

Again, estimate first:  
 $243 \times 68$  is approximately  $200 \times 70 = 14000$ .



		
Y6		
		
		
		By this time children meet 4 digits by 2 digits, the only efficient method is the standard method for Long Multiplication.
		

# Division Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

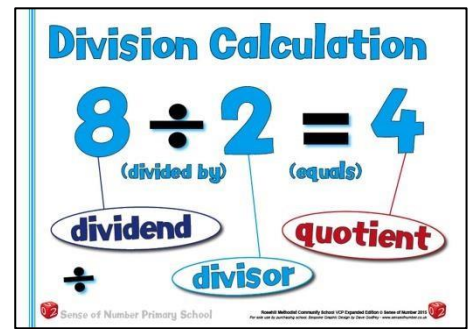
These notes show the stages in building up to long division through Years 3 to 6 – first using short division 2 digits ÷ 1 digit, extending to 3 / 4 digits ÷ 1 digit, then long division 4 / 5 digits ÷ 2 digits.

To divide successfully in their heads, children need to be able to:

- **understand and use the vocabulary of division – for example in  $18 \div 3 = 6$ , the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;**
- **recall multiplication and division facts to  $12 \times 12$ , recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;**
- **know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;**
- **understand and use multiplication and division as inverse operations.**

Children need to acquire **one efficient written method of calculation for division**, which they know they can rely on **when mental methods are not appropriate**.

**Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.**



To carry out expanded and standard written methods of division successful, children also need to be able to:

- visualise how to calculate the quotient by visualising repeated addition;
- estimate how many times one number divides into another – for example, approximately how many sixes there are in 99, or how many 23s there are in 100;
- multiply a two-digit number by a single-digit number mentally;
- understand and use the relationship between single digit multiplication, and multiplying by a multiple of 10. (e.g.  $4 \times 7 = 28$  so  $4 \times 70 = 280$  or  $40 \times 7 = 280$  or  $4 \times 700 = 2800$ .)
- subtract numbers using the column method (if using NNS 'chunking')

*For example, without a clear understanding that 72 can be partitioned into 60 and 12, 40 and 32 or 30 and 42 (as well as 70 and 2), it would be difficult to divide 72 by 6, 4 or 3 using the 'chunking' method.*

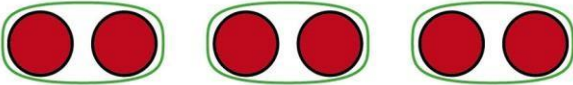
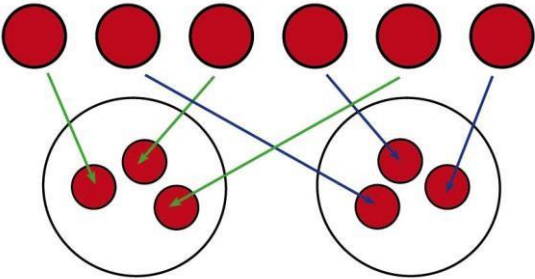
*$72 \div 6$  'chunks' into 60 and 12*

*$72 \div 4$  'chunks' into 40 and 32*

*$72 \div 3$  'chunks' into 30 and 42  
(or 30, 30 and 12)*

*The above points are crucial. If children do not have a secure understanding of these prior learning objectives then they are unlikely to divide with confidence or success, especially when attempting the 'chunking' method of division.*

Please note that there are two different 'policies' for chunking. The first would be used by schools who have adopted the NNS model, the second for schools who have made the (sensible) decision to teach chunking as a mental arithmetic / number line process, and prefer to count forwards in chunks rather than backwards.

Models	Division
<h2>Grouping</h2> <p>(The key model for division)</p>	<h3>D: Grouping</h3>  <p>"How many groups of <b>2</b> can I make out of <b>6</b>?" Answer: <b>3</b>"</p> <p><small>Sense of Number Primary School</small></p>
<h2>Sharing</h2> <p>(The model that links with fractions)</p>	<h3>D: Sharing</h3>  <p>"If I share <b>6</b> into <b>2</b> equal amounts, how many in each group?" Answer: <b>3</b>"</p> <p><small>Sense of Number Primary School</small></p>

## **Division In Key Stage 1 – Grouping or Sharing?**

**When children think conceptually about division, their default understanding should be Division is Grouping, as this is the most efficient way to divide.**

The 'traditional' approach to the introduction of division in KS1 is to begin with 'sharing', as this is seen to be more 'natural' and easier to understand.

Most children then spend the majority of their time 'sharing' counters and other resources (i.e. seeing  $20 \div 5$  as 20 shared between 5) – a rather laborious process which can only be achieved by counting, and which becomes increasingly inefficient as both the divisor and the number to be divided by (the dividend) increase)

**These children are given little opportunity to use the grouping approach.**

**(i.e.  $20 \div 5$  means how many 5's are there in 20?) – far simpler and can quickly be achieved by counting in 5s to 20, something which most children in Y1 can do relatively easily.**

**Grouping in division can also be visualised extremely effectively using number lines and Numicon.** The only way to visualise sharing is through counting.

**Grouping, not sharing, is the inverse of multiplication. Sharing is division as fractions.**

**Once children have grouping as their first principle for division they can answer any simple calculation by counting in different steps (2s, 5s, 10s then 3s, 4s, 6s etc.). As soon as they learn their tables facts then they can answer immediately.**

E.g. How much quicker can a child answer the calculations  $24 \div 2$ ,  $35 \div 5$  or  $70 \div 10$  using grouping? Children taught sharing would find it very difficult to even attempt these calculations.

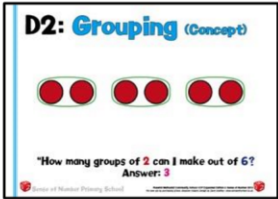
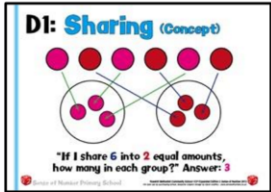

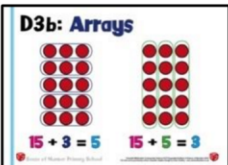
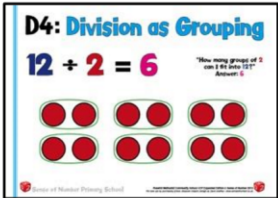
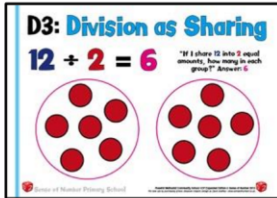
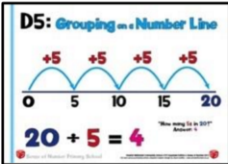
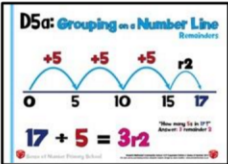
**Children who have sharing as their first principle tend to get confused in KS2 when the understanding moves towards 'how many times does one number 'go into' another'.**

**When children are taught grouping as their default method for simple division questions it means that they;**

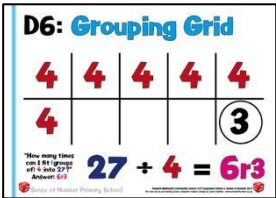
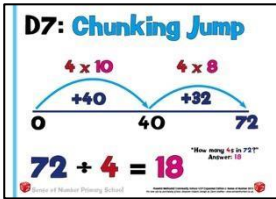
- secure understanding that the divisor is crucially important in the calculation
- can link to counting in equal steps on a number line
- have images to support understanding of what to do with remainders (Numicon)
- have a far more efficient method as the divisor increases
- have a much firmer basis on which to build KS2 division strategies

Consequently this policy is structured around the teaching of division as grouping, moving from counting up in different steps to learning tables facts and eventually progressing towards the mental chunking and 'bus stop' methods of written division in KS2.

Sharing is introduced as division in KS1, but is then taught mainly as part of the fractions curriculum, where the link between fractions and division is emphasised and maintained throughout KS2.

Stage 1	Concepts and Number Lines (pre-chunking)	
	Grouping	Sharing
FS	From EYFS onwards, children need to explore practically both <b>grouping</b> and <b>sharing</b> . Links can then be made in both KS1 and KS2 between sharing and fractions.	
Y1	Begin by giving children opportunities to use concrete objects, pictorial representations and arrays with the support of the teacher. Use the words 'sharing' and 'grouping' to identify the concepts involved. Identify the link between multiplication and division using the array image.	
		
	 	
Y2	Identify Grouping as the key model for division. Relate to knowledge of multiplication facts. Use the key vocabulary: '20 ÷ 5 means how many 5's can I fit into 20?'	Identify Sharing as the secondary model of division.
		
	Counting on is the easiest route when using a number line to solve a division calculation.	
	 	
Y3	Continue to give children practical images for division by grouping: e.g. use PE mats and ask children to move into groups of 4. The remainder go into a hoop. Use Numicon shapes – how many 4 pieces can I fit into 27 (made of two tens and a seven piece).	<div> <p><b>Regularly stress the link between multiplication and division, and how children can use their tables facts to divide by counting forwards in steps.</b></p> </div>



		
Stage 2	Chunking & Standard Methods	
	Chunking Find the Hunk & NNS Chunking	Standard Methods
	As previously encountered in Y2, developing an understanding of division with the number line is an excellent way of linking division to multiplication. It can show division both as repeated subtraction, but it is simpler to show division by counting forward to find how many times one number 'goes into' another.	
Y3		
	<p>'Find the Hunk' is a mental strategy based on mental partitioning. For the example below, the Hunk is defined as being 10 times the divisor. i.e. the divisor is 4, so the Hunk will be <math>4 \times 10 = 40</math>. Both chunks are then divided by the divisor and then the groups totaled.</p> <p>Where as 'Find the Hunk' is a mental strategy based on mental partitioning, the National Strategy chunking method is based on subtraction. Here 40 (<math>4 \times 10</math>) is initially subtracted from the dividend. This strategy is somewhat confusing and the recommendation is to use Find the Hunk as the default strategy.</p>	<p>These slides introduce the Short Division (Bus Stop) method in Year 3. It is recommended that if children are introduced to this strategy in Year 3, it is only introduced at the end of Year 3 (ideally kept until Year 4) and that the key methods in Year 3 remain the use of Number Lines and the mental chucking method known as 'Find the Hunk' (see opposite)</p> <p>When introducing Short Division formally, use dienes (Base 10) and make sure you introduce it using the <b>sharing model</b>. The calculation starts with, 'I have 7 tens, to share between 4 people. That's 1 each with 3 remaining. These three tens are exchanged into 30, ones. The 32 ones are now shared between 4 people – that's 8, ones each.'</p>

	<div data-bbox="365 98 643 297"> <p><b>D8: Find the Hunk!</b></p> <p><math>72 \div 4 = 18</math></p> <p>The Hunk! <math>40 + 32</math></p> <p><math>\downarrow \quad \downarrow \quad \div 4</math></p> <p><math>10 + 8 = 18</math></p> </div> <div data-bbox="660 98 925 297"> <p><b>(D11: Chunking)</b></p> <p><math>4 \overline{)72}</math></p> <p><math>-40 \ (4 \times 10)</math></p> <p><math>-32 \ (4 \times 8)</math></p> <p><math>0</math></p> <p><math>72 \div 4 = 18</math></p> </div>	<div data-bbox="1088 98 1361 297"> <p><b>(D10: Short Division)</b></p> <p><math>72 \div 4 = 18</math></p> <p><math>4 \overline{)72}</math></p> </div>
	<p>Show the children examples of chunking where the quotient includes remainders.</p>	
	<div data-bbox="413 409 643 573"> <p><b>D7a: Chunking Jump</b></p> <p><math>4 \times 10 \quad 4 \times 6 \quad r1</math></p> <p><math>0 \quad +40 \quad +24 \quad 65</math></p> <p><math>65 \div 4 = 16r1</math></p> </div> <div data-bbox="660 409 877 573"> <p><b>D8a: Find the Hunk!</b></p> <p><math>65 \div 4 = 16r1</math></p> <p>The Hunk! <math>40 + 25</math></p> <p><math>\downarrow \quad \downarrow \quad \div 4</math></p> <p><math>10 + 6r1 = 16r1</math></p> </div> <div data-bbox="528 582 798 770"> <p><b>(D11: Chunking)</b></p> <p><math>4 \overline{)65}</math></p> <p><math>-40 \ (4 \times 10)</math></p> <p><math>25</math></p> <p><math>-24 \ (4 \times 6)</math></p> <p><math>1</math></p> <p><math>65 \div 4 = 16r1</math></p> </div>	<div data-bbox="1088 409 1361 604"> <p><b>(D10: Short Division)</b></p> <p><math>65 \div 4 = 16r1</math></p> <p><math>4 \overline{)65}</math></p> </div>
	<p>'Mega Hunk' is the natural development of the 'Find the Hunk' strategy. Here Mega Hunk is defined as being multiple of 10 times the divisor. In the case below the divisor is 4, so the Hunk will be <math>4 \times (10 \times 3) = 120</math>. Again, both chunks are then divided by the divisor and then the groups totaled.</p> <p>The National Strategy chunking method is also based on the multiples of 10 times the divisor. D11b slide is an expanded version of D11. Jottings can be made to spot the multiples of 10 times the divisor (e.g. 40, 80, 120 etc.). This strategy links to the <b>Grouping model</b>.</p>	
Y4	<div data-bbox="507 1254 785 1451"> <p><b>D9: Mega Hunk!</b></p> <p><math>136 \div 4 = 34</math></p> <p>Mega Hunk! <math>120 + 16</math></p> <p><math>\downarrow \quad \downarrow \quad \div 4</math></p> <p><math>30 + 4 = 34</math></p> </div> <div data-bbox="413 1467 643 1630"> <p><b>D11: Chunking</b></p> <p><math>4 \overline{)136}</math></p> <p><math>-120 \ (4 \times 30)</math></p> <p><math>16</math></p> <p><math>-16 \ (4 \times 4)</math></p> <p><math>0</math></p> <p><math>136 \div 4 = 34</math></p> </div> <div data-bbox="660 1467 877 1630"> <p><b>D11b: Chunking</b></p> <p><math>4 \overline{)136}</math></p> <p><math>-40 \ (4 \times 10)</math></p> <p><math>96</math></p> <p><math>-40 \ (4 \times 10)</math></p> <p><math>56</math></p> <p><math>-40 \ (4 \times 10)</math></p> <p><math>16</math></p> <p><math>-16 \ (4 \times 4)</math></p> <p><math>0</math></p> <p><math>136 \div 4 = 34</math></p> </div>	<div data-bbox="1088 1400 1361 1594"> <p><b>D10: Short Division</b></p> <p><math>136 \div 4 = 34</math></p> <p><math>4 \overline{)136}</math></p> </div>
Y5	<p>Continue to use the Find the Hunk strategy whenever possible.</p>	
	<div data-bbox="375 1818 643 2007"> <p><b>D9c: Mega Hunk!</b></p> <p><math>394 \div 6 = 65r4</math></p> <p>Mega Hunk! <math>360 + 34</math></p> <p><math>\downarrow \quad \downarrow \quad \div 6</math></p> <p><math>60 + 5r4 = 65r4</math></p> </div> <div data-bbox="660 1818 917 2007"> <p><b>D11c: Chunking</b></p> <p><math>6 \overline{)394}</math></p> <p><math>-360 \ (6 \times 60)</math></p> <p><math>34</math></p> <p><math>-30 \ (6 \times 5)</math></p> <p><math>4</math></p> <p><math>394 \div 6 = 65r4</math></p> </div>	<div data-bbox="1088 1818 1361 2013"> <p><b>D10c: Short Division</b></p> <p><math>394 \div 6 = 65r4</math></p> <p><math>6 \overline{)394}</math></p> </div>

	<div>D9d: <b>Mega Hunk!</b>  <math>591 \div 3 = 197</math>  Mega Hunk! Mega Hunk! Chunk  <math>300 + 270 + 21 \div 3</math>  <math>100 + 90 + 7 = 197</math></div> <div>D11d: <b>Chunking</b>  Mega Chunk  <math>197</math>  <math>3 \overline{)591}</math>  <math>-300 (3 \times 100)</math>  <math>291</math>  <math>-270 (3 \times 90)</math>  <math>21</math>  <math>-21 (3 \times 7)</math>  <math>0</math>  <math>591 \div 3 = 197</math></div>	<div>D10d: <b>Short Division</b>  <math>591 \div 3 = 197</math>  <math>3 \overline{)591}</math></div>
	<div>D9e: <b>Mega Hunk!</b>  <math>5978 \div 7 = 854</math>  Mega Hunk! Mega Hunk! Chunk  <math>5600 + 350 + 28 \div 7</math>  <math>800 + 50 + 4 = 854</math></div> <div>D11e: <b>Chunking</b>  Mega Chunk  <math>854</math>  <math>7 \overline{)5978}</math>  <math>-5600 (7 \times 800)</math>  <math>378</math>  <math>-350 (7 \times 50)</math>  <math>28</math>  <math>-28 (7 \times 4)</math>  <math>0</math>  <math>5978 \div 7 = 854</math></div> <div>D10e: <b>Short Division</b>  <math>5978 \div 7 = 854</math>  <math>7 \overline{)5978}</math></div>	
	<div>Begin by subtracting several chunks, but then try to find the biggest chunks of the divisor that can be subtracted.</div>	
		<div>Children should develop the ability to represent the quotient to include a straight forward remainder, but also as a decimal or fractional remainder.</div>
	<div>D9f: <b>Mega Hunk!</b>  <math>846 \div 5 = 169r1</math>  Mega Hunk! Mega Hunk! Chunk  <math>500 + 300 + 46 \div 5</math>  <math>100 + 60 + 9r1 = 169r1</math></div> <div>D11f: <b>Chunking</b>  Mega Chunk  <math>169r1</math>  <math>5 \overline{)846}</math>  <math>-500 (5 \times 100)</math>  <math>346</math>  <math>-300 (5 \times 60)</math>  <math>46</math>  <math>-45 (5 \times 9)</math>  <math>1</math>  <math>846 \div 5 = 169r1</math></div>	<div>D10f: <b>Short Division</b>  Different Remainders  <math>846 \div 5</math>  <math>5 \overline{)846.0}</math>  <math>169.2</math>  <math>5 \overline{)846}</math>  <math>169r1</math>  <math>5 \overline{)846}</math>  <math>169\frac{1}{5}</math></div>
<b>Y6</b>	<div>When introducing long division, it is often easier to find the quotient using the Mega Hunk strategy.</div>	
	<div>D9g: <b>Mega Hunk!</b>  Simple Long Division  <math>480 \div 15 = 32</math>  Mega Hunk! Chunk  <math>450 + 30 \div 15</math>  <math>30 + 2 = 32</math></div>	
	<div>D11g1: <b>Chunking</b>  Long Division  <math>32</math>  <math>15 \overline{)480}</math>  <math>-450 (15 \times 30)</math>  <math>30</math>  <math>-30 (15 \times 2)</math>  <math>0</math>  <math>480 \div 15 = 32</math></div> <div>D11g2: <b>Chunking</b>  Long Division  <math>32</math>  <math>15 \overline{)480}</math>  <math>-150 (15 \times 10)</math>  <math>330</math>  <math>-150 (15 \times 10)</math>  <math>180</math>  <math>-150 (15 \times 10)</math>  <math>30</math>  <math>-30 (15 \times 2)</math>  <math>0</math>  <math>480 \div 15 = 32</math></div>	
	<div>D9h: <b>Decimal Hunk!</b>  <math>18 \div 1.5 = 12</math>  The Hunk! Chunk  <math>15 + 3 \div 1.5</math>  <math>10 + 2 = 12</math></div> <div>D9i: <b>Decimal Hunk!</b>  <math>87.5 \div 7 = 12.5</math>  Mega Hunk! Chunk  <math>70 + 14 + 3.5 \div 7</math>  <math>10 + 2 + 0.5 = 12.5</math></div>	<div>D10i: <b>Short Division</b>  <math>87.5 \div 7 = 12.5</math>  <math>7 \overline{)87.5}</math></div>
	<div>There are three different ways of calculating using Long Division: The Short Division method, the Traditional Method and the NNS Chunking method. The Traditional Long Division method ignores place value, and therefore is not as helpful as the Chunking Method, which now becomes the recommended strategy.</div>	

	<div data-bbox="365 105 643 297"> <p><b>D13: Long Division</b> Chunking Method</p> <math display="block">\begin{array}{r} 26r21 \\ 37 \overline{) 983} \\ \underline{- 740} \quad (37 \times 20) \\ 243 \\ \underline{- 222} \quad (37 \times 6) \\ 21 \end{array}</math> <p><math>983 \div 37 = 26r21</math></p> </div> <div data-bbox="660 105 925 297"> <p><b>D13j: Long Division</b> Chunking Method</p> <math display="block">\begin{array}{r} 26r21 \\ 37 \overline{) 983} \\ \underline{- 370} \quad (37 \times 10) \\ 613 \\ \underline{- 370} \quad (37 \times 10) \\ 243 \\ \underline{- 222} \quad (37 \times 6) \\ 21 \end{array}</math> <p><math>983 \div 37 = 26r21</math></p> </div>	<div data-bbox="994 105 1219 264"> <p><b>D12: Long Division</b> Short Division Method</p> <math display="block">\begin{array}{r} 26r21 \\ 37 \overline{) 983} \end{array}</math> </div> <div data-bbox="1236 105 1460 264"> <p><b>D14: Long Division</b> Traditional Method</p> <math display="block">\begin{array}{r} 26r21 \\ 37 \overline{) 983} \\ \underline{- 74} \\ 243 \\ \underline{- 222} \\ 21 \end{array}</math> <p><math>983 \div 37 = 26r21</math></p> </div>
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