Friezland Primary School
Calculation Policy

Adopted: January 2019

Original document written and prepared by Anthony Reddy, Dave Godfrey and Laurence Hicks

Sense of Number Maths Consultants

<u>Contents:</u>

Aims and Rationale

The need to have a coherent, progressive calculation policy, which is understood and used consistently throughout the school and by parents and carers, is of great importance to us at Friezland School.

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas, they develop ways of:

- Recording to support their thinking and calculation methods
- Using particular methods that apply to special cases
- Interpreting and using the signs and symbols involved

As children's methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and lead to formal written methods that can be used more generally.

Early practical, oral and mental work *must lay the foundations* by providing children with a good understanding of:

- How the four operations build on efficient counting strategies
- Place value
- Number facts

When children leave primary school we aim to ensure that they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and other strategies to cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- apply efficient strategies to problem solving and real life situations;
- have strategies to allow them to check if their answers are realistic and accurate.

Overview of Calculation Approaches

Early Years into KS1

- Visualisation to secure understanding of the number system, especially the use of place value resources such as Base 10 and 100 Squares.
- Secure understanding of numbers to 10, using resources such as Tens Frames, fingers and multi-link.
- To begin making links between the different images of a number and their links to calculation.
- Practical, oral and mental activities to understand calculation.
- Personal methods of recording.

Key Stage 1

- Introduce signs and symbols ***(+ , -, x, ÷ in Year 1 and <, > signs in Year 2)***
- Extended visualisation to secure understanding of the number system beyond 100, especially the use of place value resources such as Base 10, Place Value Charts & Grids, Number Grids, Arrow Cards and Place Value Counters.
- Further work on recognising numbers without counting and Tens Frames to develop basic calculation understanding, supported by multi-link.
- Continued use of practical apparatus to support the early teaching of 2-digit calculation. For example, using base 10 to demonstrate partitioning and exchanging before these methods are taught as jottings / number sentences.
- Methods of recording / jottings to support calculation (e.g. partitioning or counting on).
- Use images such as empty number lines to support mental and informal calculation.

Year 3

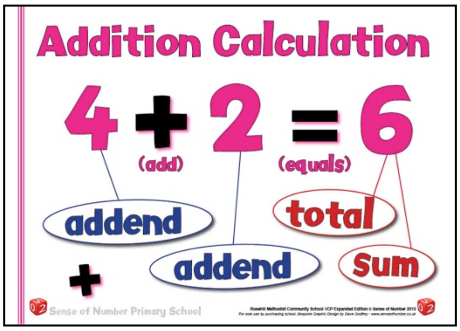
- Continued use of practical apparatus, especially Place Value Counters and Base 10 to visualise written / column methods before and as they are actually taught as procedures.
- Continued use of mental methods and jottings for 2 and 3 digit calculations.
- Introduction to more efficient informal written methods / jottings including expanded methods and efficient use of number lines (especially for subtraction).
- Column methods, where appropriate, for 3 digit additions and subtractions.

Years 3-6

- Continued use of mental methods for any appropriate calculation up to 6 digits.
- Standard written (compact) / column procedures to be learned for all four operations
- Efficient informal methods (expanded addition and subtraction, grid multiplication, division by chunking) and number lines are still used when appropriate. Develop these to larger numbers and decimals where appropriate.

N.B. Children must still be allowed access to practical resources to help visualise certain calculations, including those involving decimals

Addition Progression



The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Children need to acquire **one efficient written method of calculation** for addition that they know they can rely on **when mental methods are not appropriate.**

To add successfully, children need to be able to:

- **recall all addition pairs to 9 + 9 and complements in 10;**
- **add mentally a series of one-digit numbers, such as 5 + 8 + 4;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

Mental Addition Methods

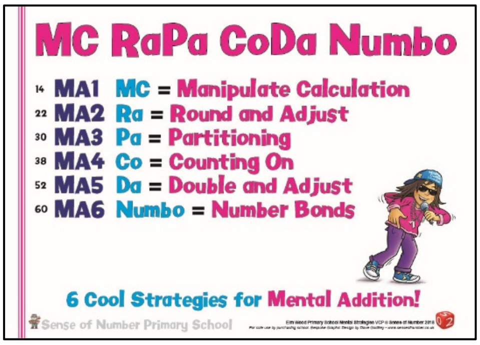
There are **6 key mental methods** for addition, which need to be a regular and consistent part of the approach to calculation in all classes from Year 2 upwards.

These methods will be introduced individually when appropriate, and then be rehearsed and consolidated throughout the year until they are almost second nature.

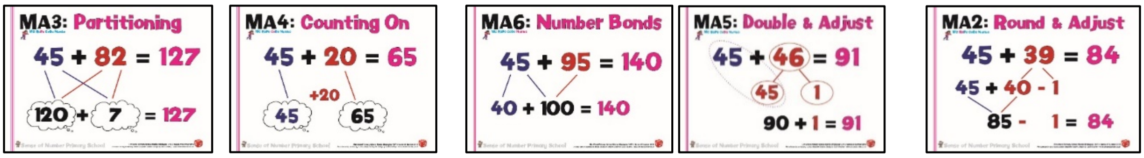
These methods are **Partitioning, Counting On, Manipulate the Calculation, Round & Adjust, Double & Adjust** and using **Number Bonds.** The first two methods are also part of the written calculation policy (see pages 14-18) but can equally be developed as simple mental calculation methods once children are skilled in using them as jottings.

Using the acronym **MC RAPA CODA NUMBO**, children can be given weekly practice in choosing the most appropriate method whenever they are faced with a simple addition, usually of 2 or 3-digit numbers, but also spotting the opportunities when they can be used with larger numbers (E.g. 3678 + 2997) or decimals (E.g. 4.8 + 2.2)

MC	**Manipulate Calculation**
RA	**Round & Adjust**
PA	**Partitioning**
CO	**Counting On**
DA	**Double & Adjust**
NUMBO	**Number Bonds**



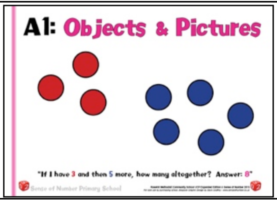
For example, using the number 45 (see examples below), we can look at the other number chosen, and decide on the most appropriate mental calculation method.

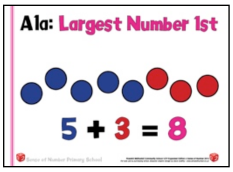


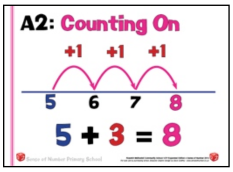
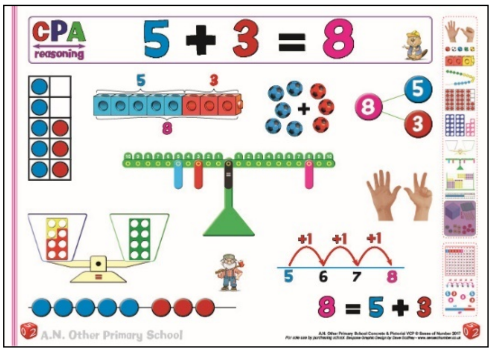
These 5 methods need to be taught to the children so that they can select the most appropriate method depending on the numbers in the sum.

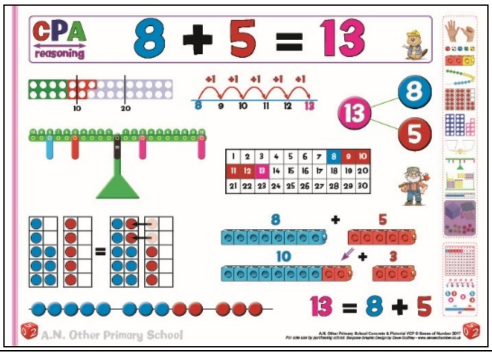
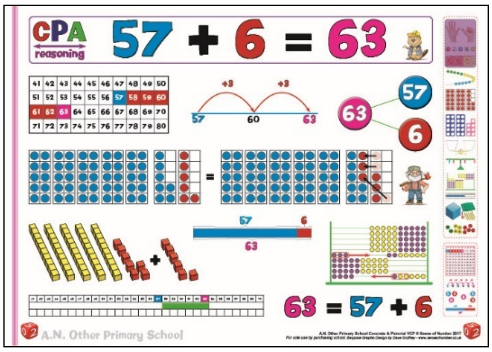
Below you can see the progression of each method through the year groups, with some appropriate examples of numbers, which may be used for method.

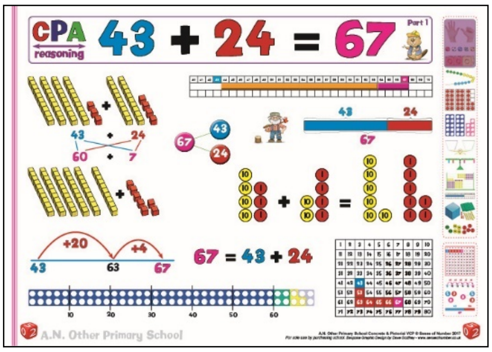
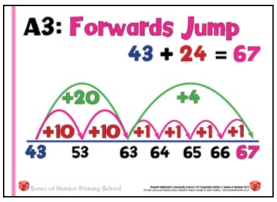
Written Methods of Addition

<table>
<tr><th>Stage 1</th><th>Finding a Total and the Empty Number Line</th><th>Alternative Method:
Counting on Mentally or as a jotting</th></tr>
<tr><td>FS/Y1</td><td>Initially, children need to represent addition using a range of different resources and understand that a total can be found by counting out the first number, counting out the second number then counting how many there are altogether.</td><td></td></tr>
<tr><td></td><td>

Recognise counting can be done in either order (3+5 or 5+3)</td><td>3 (held in head) then use fingers to count on 5 (“3… 4,5,6,7,8)</td></tr>
<tr><td></td><td>This will quickly develop into placing the largest number first, either as a pictorial / visual method or by using a number line. The use of the number line, however, should be delayed until the children are completely secure in their understanding of 5 + 3. Otherwise it becomes a tool that limits their understanding of what they are actually representing



</td><td>5 (held in head) then count on 3 (“5 … 6, 7, 8”)</td></tr>
<tr><td></td><td>Before moving onto jottings or written methods (for any calculation), children need to be shown a wide range of images that support their understanding.
For simple calculations, the ‘equals’ sign can be viewed as a way to show the total (egg boxes, cubes, footballs, fingers) but also as a ‘balance’ (peg balance), where 5 and 3 have a value that is equal to / the same as, or that balances 8.

</td><td></td></tr>
</table>

<table>
<tr>
<td>Y1/2</td>
<td>The next stage is bridging through 10 using practical resources.

</td>
<td>Once this image is secure, the same methods can be done mentally: -
8 (in head) then count on 5
(“8 … 9, 10, 11, 12, 13”)
Or “8 + 2 = 10…
10 + 3 = 13).</td>
</tr>
<tr>
<td></td>
<td>The next step is to bridge through a multiple of 10.

</td>
<td>Again, the number line jotting can display counting on (57+1+1+1+1+1+1) or partitioning (57 + 3 + 3)

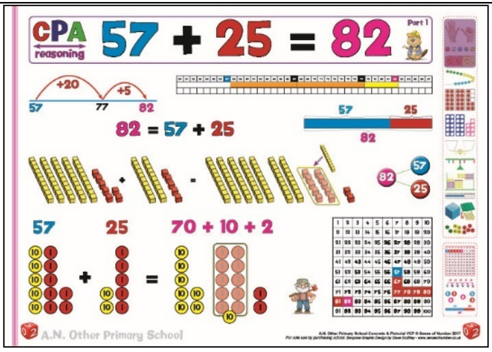
This picture then becomes the mental methods: -
57 (in head) - count on 6
(“57, 58,59,60,61,62,63”)
Or “57 + 3 = 60 …
60 + 3 = 63)</td>
</tr>
<tr>
<td></td>
<td>The next stage is to add 2 digit numbers without bridging 10

</td>
<td>Firstly, add the tens then the ones individually


(43 + 24 =
43+10+10+1+1+1+1)
before counting on in tens and ones (43 + 20 = 63 ..
63 + 4 = 67)</td>
</tr>
<tr>
<td></td>
<td>Develop to crossing the 10s, then the 100s boundary
57 + 25 = 82   86 + 48 = 134</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Even as the calculations increase, and start to deal with exchanging, it is important that they are initially visualised using a range of resources / images.</td>
<td>By this stage the number line no longer counts in individual 10s & 1s.

57 + 25
57 + 20 = 77…
77 + 5 = 82

Extend to crossing the 100s

86 + 48</td>
</tr>
</table>



86 + 40 = 126

126 + 8 = 134

Y3/4

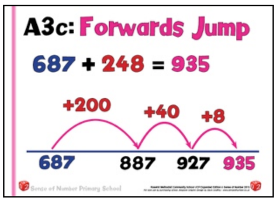
For some children, the number line method can still be used for 3-digit calculations.

687 + 200 = 887…

887 + 40 = 927…

927 + 8 = 935

Or 687 + 200 + 40 + 8 = 935



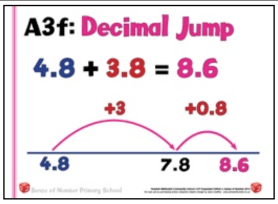
Y5/6

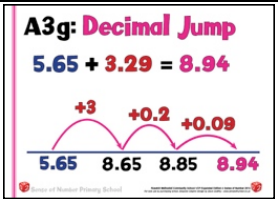
In Years 5 and 6, if necessary, children can return to the number line method to support their understanding of decimal calculation

4.8 + 3 = 7.8

7.8 + 0.8 = 8.6

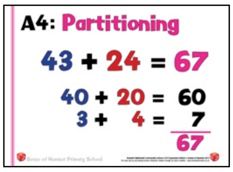
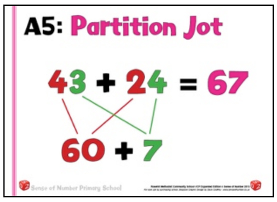
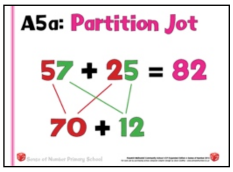
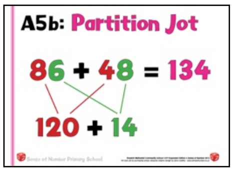
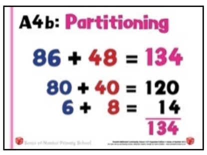
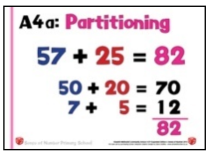
Or 4.8 + 3 + 0.8 = 8.6

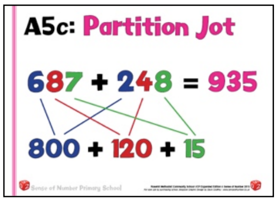
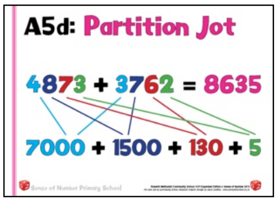
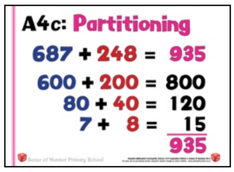
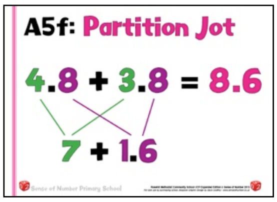
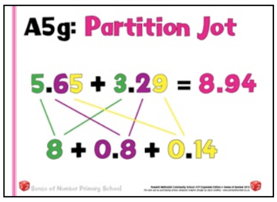


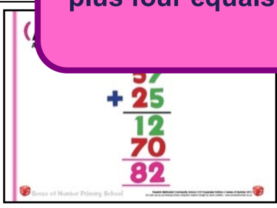
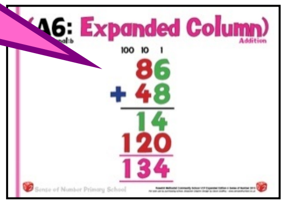


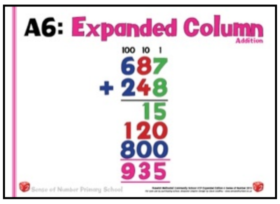
Number lines support children's thinking if they find partitioning / column addition difficult, as it simply involves counting on in 100s, 10s & 1s.

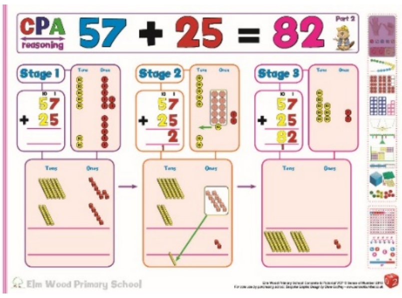
Hopefully, with the above calculation, many children would mentally Round & Adjust (4.8 + 4 – 0.2 = 8.6)

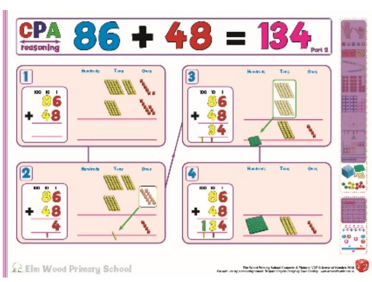
Stage 2	Partition Jot	Alternative Method: Traditional Partitioning
Y2/3	**Traditionally, partitioning has been presented using the method on the right. Although this does support place value and the use of arrow cards, it is very laborious, so it is suggested that adopting the 'partition jot' method will improve speed and consistency for mental to written (or written to mental) progression**	Record steps in addition using partition, initially as a jotting: - **43 + 24 =** 40 + 20 + 3 + 4 = 60 + 7 = 67 Or, preferably 
	As soon as possible, refine this method to a much quicker and clearer '**Partition Jot**' approach 	
	As before, develop these methods, especially Partition Jot, towards crossing the 10s and then 100s.	
	 	 

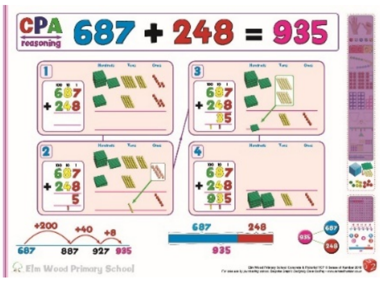
	This method will soon become the recognised jotting to support the teaching of partitioning. It can be easily extended to 3 and even 4-digit numbers when appropriate.	For certain children, the traditional partitioning method can still be used for 3-digit numbers, but it is probably too laborious for 4-digit numbers.
Y3/4	 	
	Partition jot is also extremely effective as a quicker alternative to column addition for decimals.	Some simple decimal calculations can also be completed this way.
Y5/6	 	

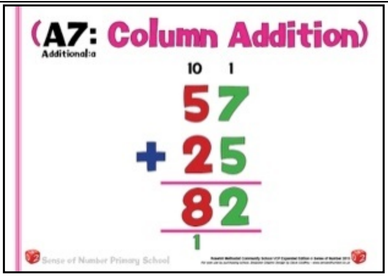
Stage 3	**Expanded Method in Columns**	
Y3	**Column methods of addition are introduced in Year 3, but it is crucial that they still see mental calculation as their first principle, especially for 2-digit numbers.** **Column methods should only be used for more difficult calculations, usually with 3-digit numbers that cross the Thousands boundary or most calculations involving 4-digit numbers and above.** **N.B. Even when dealing with bigger numbers / decimals, children should still look for the opportunity to calculate mentally (E.g. 4675 + 1998)**	
	2 digit examples are used below simply to introduce column methods to the children. Most children would continue to answer these calculations mentally or using a simple jotting.	
	Using the column, children need to learn the principle of adding the ones first rather than the tens.	
	The 'expanded' method is a very effective introduction to column addition. It continues to use the partitioning method that the children are already familiar with, but begins to set out calculations vertically. It is particularly helpful for automatically 'dealing' with the 'carry' digit. **It is crucial, however, to ensure that practical apparatus has been used first before any sort of columnar procedure is introduced (see examples below in the 'Column Method' section**	
Y3/4	**A. Single 'carry' in ones**	**B. 'Carry' in ones and tens**
		**'Eighty plus forty equals one hundred and twenty, because 'eight plus four equals twelve.** 
	Once this method is understood, it can quickly be adapted to using with three-digit numbers. It is rarely used for 4 digits and beyond as it becomes too unwieldy.	

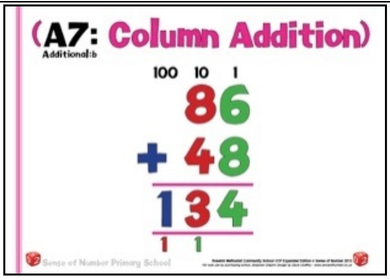
<table>
<tr><td></td><td></td></tr>
<tr><td></td><td>
</td></tr>
<tr><td></td><td>The time spent on practicing the expanded method will depend on security of number facts recall and understanding of place value.
Once the children have had enough experience in using expanded addition and have also used practical resources (Base 10 / place value counters) to model exchanging in columns, they can be taken on to standard, ‘traditional’ column addition.</td></tr>
</table>

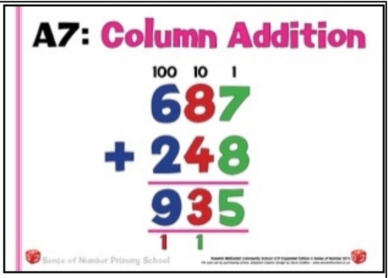
<table>
<tr><th>Stage 4</th><th>Column Method</th></tr>
<tr><td>Y3/4</td><td>As with the expanded method, begin with 2-digit numbers, simply to demonstrate the method, before moving to 3-digit numbers.
Make it <u>very clear</u> to the children that they are still expected to deal with all 2-digit (and many 3 digit) calculations mentally (or with a jotting), and that the column method is designed for numbers that are too difficult to access using these ways. The column procedure <u>is not</u> intended for use with 2-digit numbers unless completely necessary for certain children who have no other method.</td></tr>
<tr><td></td><td>‘Carry’ ones then ones and tens.
The most important part of the transition from mental and jotting approaches towards the use of column methods is the initial use of concrete materials to make / create and discuss the understanding behind the method. If children have had regular experience of using Base 10 apparatus and then Place Value Counters to make and solve calculations, (especially the regrouping of 10 ‘ones’ into 1 ‘Ten’ and 10 ‘Tens’ into 1 ‘Hundred’) then the column method is simply a written version of what they already understand.


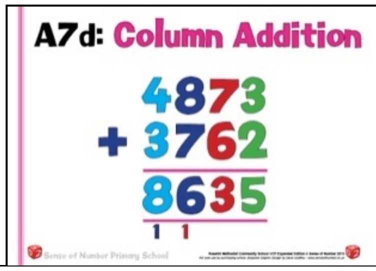


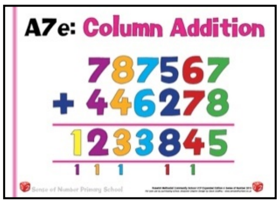

</td></tr>
</table>

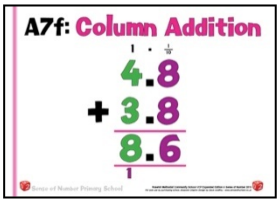
<table>
<tr>
<td></td>
<td>


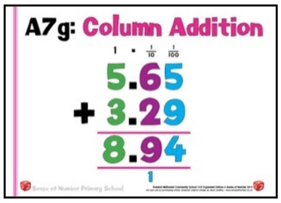


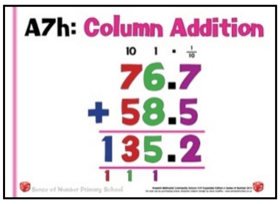


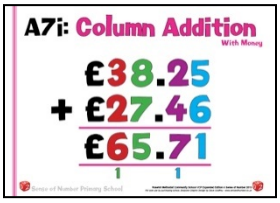
</td>
</tr>
<tr>
<td>Y4</td>
<td>Once confident, use the column method with 4-digit numbers. At this point the apparatus is not necessary as calculation procedure should be fully understood.


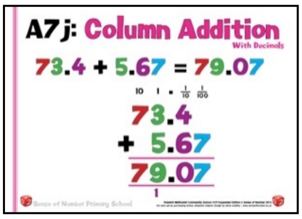
</td>
</tr>
<tr>
<td>Y5/6</td>
<td>Extend column methods to 5/6-digit calculations then decimal calculations (Year 5).</td>
</tr>
<tr>
<td colspan="2">If children make repeated errors at any stage, they can return to the expanded method or an earlier jotting.










</td>
</tr>
<tr>
<td></td>
<td>The key skill in upper Key Stage 2 that needs to then be developed, usually in Year 6, is the laying out of the column method for calculations with decimals in different places.</td>
</tr>
<tr>
<td></td>
<td>


</td>
</tr>
</table>

Subtraction Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

To subtract successfully, children need to be able to:

- **recall all addition and subtraction facts to 20;**
- **subtract multiples of 10 (such as 160 – 70) using the related subtraction fact (e.g. 16 – 7), and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).**

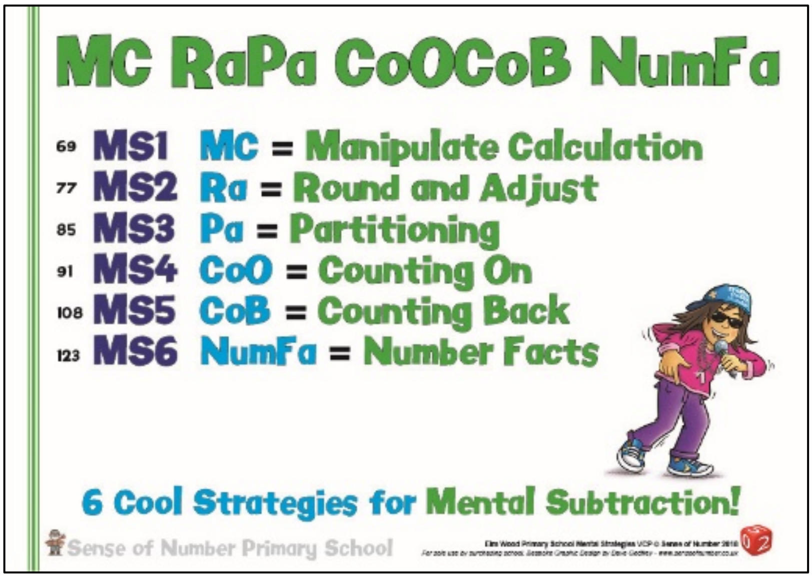
Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Mental Subtraction Strategies

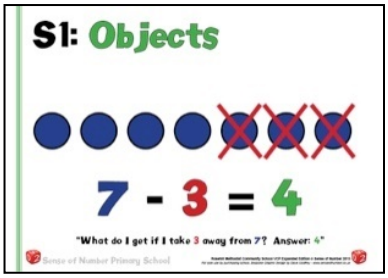
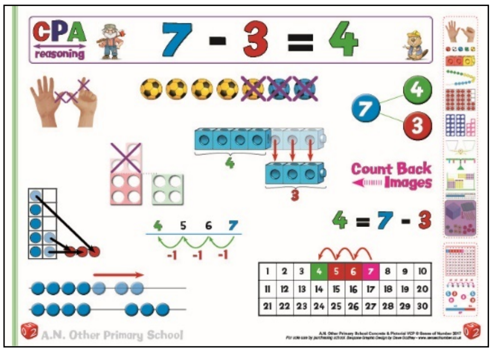
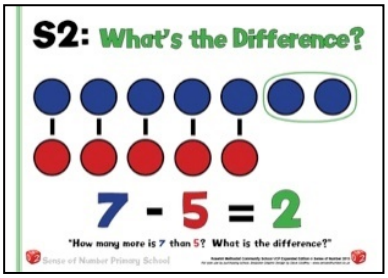
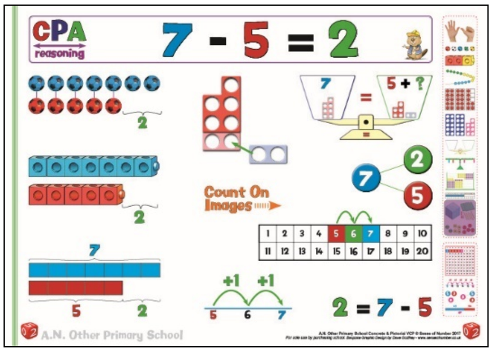
There are **6 key mental strategies** for subtraction, which need to be a regular and consistent part of the approach to calculation in all classes from Year 2 upwards.

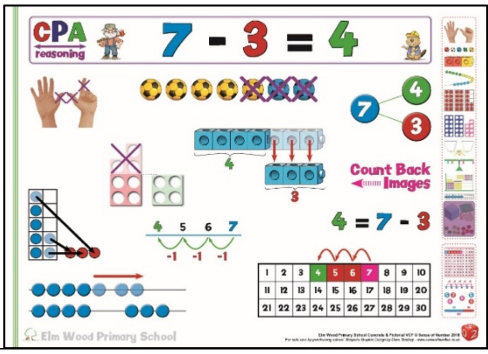
These strategies will be introduced individually when appropriate, and then be rehearsed and consolidated throughout the year until they are almost second nature.

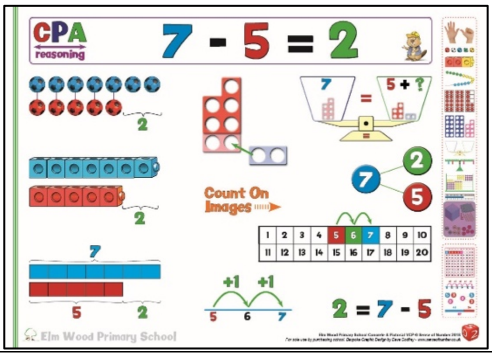
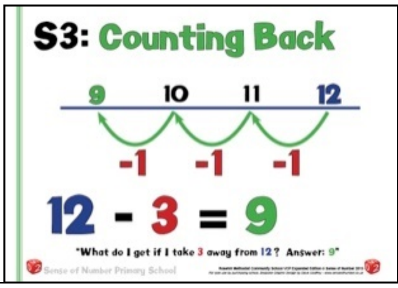
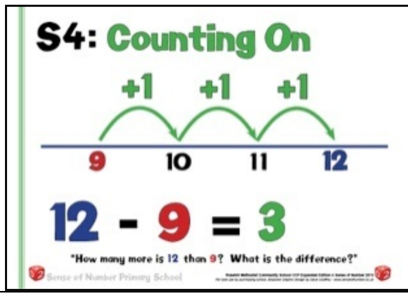
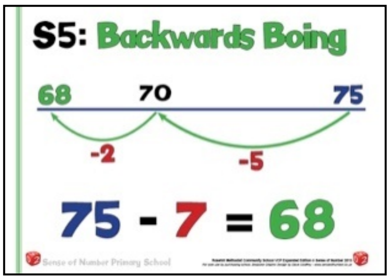
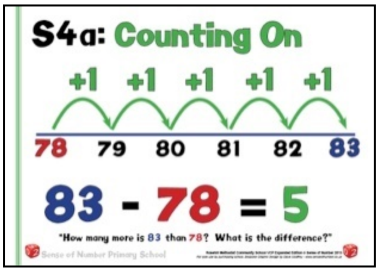
These strategies are **Counting On, Counting Back, Partitioning, Manipulate the Calculation, Round & Adjust** and using **Number Facts.**

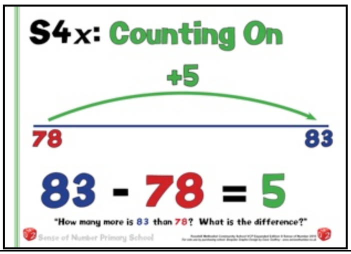
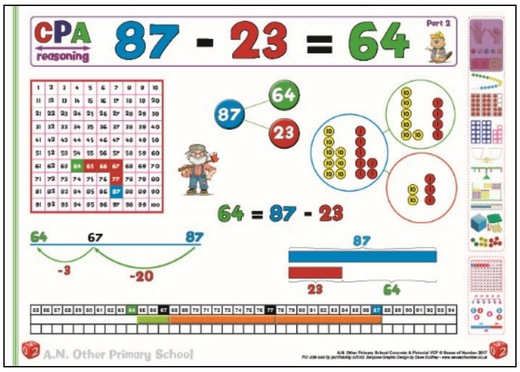
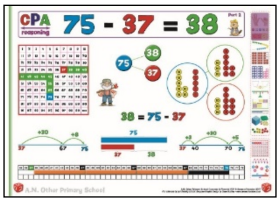


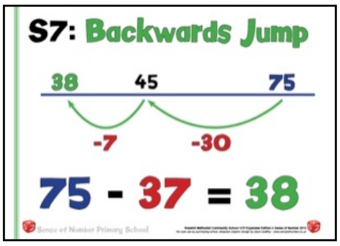
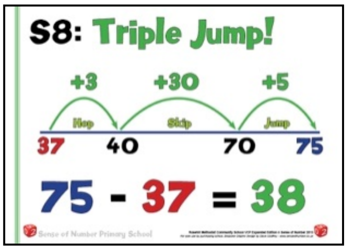
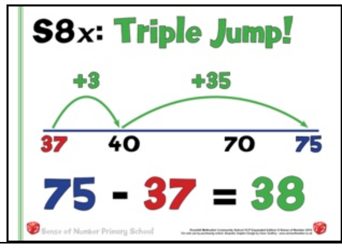
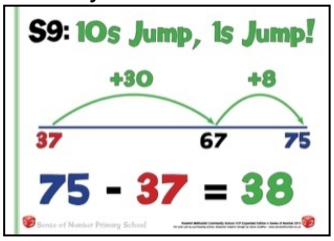
Written Methods of Subtraction

INTRO	Subtraction by counting back (or taking away)	Subtraction by counting up (or complementary addition)
FS/Y1	Early subtraction in EYFS will primarily be concerned with '***taking away'***, and will be modelled using a wide range of models and resources. These will usually be natural resources and real-life objects, and will often be part of a story telling scenario where the children can 'make' subtraction to tell the story	
	This will continue in Year 1, using resources such as multi-link cubes, Ten Frames, bead strings and pictures (like the footballs shown below) to model the 'take away' approach to subtraction. Once this is secure then images such as the desktop number track / line can be used to practice taking away practically, and then developed into counting back on demarcated number lines.  	In Year 1, it is also vital that children understand the concept of subtraction as 'finding a difference' by **comparison** and realise that **<u>any</u>** subtraction can be answered in 2 different ways, either by counting up or counting back. Again, this needs to be modelled and consolidated regularly using a wide range of resources, especially multilink towers, counters. The images below also show the early version of bar modelling, as well as the Numicon pieces being used to demonstrate the equals sign as a balance (7 is equal to 5 + __  

<table>
<tr><th rowspan="2">Stage 1</th><th colspan="2">Using the empty number line</th></tr>
<tr><th>Subtraction by counting back
(or taking away)</th><th>Subtraction by counting up
(or complementary addition)</th></tr>
<tr><td></td><td colspan="2">Before developing any jottings, mental methods or column procedures, children need to explore, discuss and visualize the two main models of subtraction using a wide range of models, images and apparatus. (including, multi-link cubes and Tens Frames). It is not advised to begin using a number line as a tool for calculation until the children understand the principles behind why it is being used. Otherwise the number line becomes a method, rather than a tool to embed and further understanding and fluency.
Images for 'counting back' Images for 'counting on



</td></tr>
<tr><td>Y1</td><td colspan="2">The empty number line helps to record or explain the steps in mental subtraction.
It is an ideal model for counting back and bridging ten, as the steps can be shown clearly.
It can also show counting up from the smaller to the larger number to find the difference.</td></tr>
<tr><td></td><td>The steps often bridge through a multiple of 10.
12 – 3 = 9

</td><td>Small differences can be found by counting up
12 – 9 = 3

</td></tr>
<tr><td>Y2/3</td><td>This is developed into crossing any multiple of 10 boundary.
75 – 7 = 68

</td><td>For 2 (or 3) digit numbers close together, count up. First, count in ones
83 – 78 = 5

</td></tr>
</table>

<table>
<tr>
<td></td>
<td></td>
<td>Then, use number facts to count up in a single jump
</td>
</tr>
<tr>
<td></td>
<td>For 2-digit numbers, count back in 10s and 1s. As with addition, it is important to let the children manipulate materials in order to ‘see’ the calculation. Base 10 and Place Value Counters are particularly crucial as these are images which are used throughout the topic (see expanded method section for these images). The image below uses Place Value Counters to demonstrate the Number Bond diagram. Once secure, model the use of the empty number line where tens and ones are subtracted in single jumps (87 – 20 – 3)
</td>
<td></td>
</tr>
<tr>
<td></td>
<td colspan="2">Some numbers (75 – 37) can be subtracted just as quickly either way.
The images below show 75-37 as a Number Bond Diagram (also visualised with Place Value Counters), as a ‘count up’ image using the number line (explored further below in 2 different ways) and number rods, and as a take away / count back image on the 100 Square.

The number line itself is an excellent visual jotting for both counting on and counting back, depending on the children’s preferred method: -</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td>Either count back 30 then count back 7
</td>
<td>Or count up from smaller to the larger number, initially with a ‘triple jump’ method of jumping to the next 10, then multiples of 10, then to the target number.

This can also be done in 2 jumps.
</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Some children prefer to jump in tens and ones, which is an equally valid method, as it links to the mental skill of ‘counting up from any number in tens’
</td>
</tr>
</table>

Stage 2 — Expanded Method & Number Lines (continued)

Subtraction by counting back Expanded Method	Subtraction by counting up Number Lines (continued)

In Year 3, according to the New Curriculum, children are expected to be able to use both jottings <u>and</u> written column methods to deal with 3-digit subtractions.

This is only guidance, however – as long as children leave Year 6 able to access all four operations using formal methods, schools can make their own decisions as to when these are introduced.

It is very important that they have had regular opportunities to use the number line 'counting up' approach first (right hand column below) so that they already have a secure method that is almost their first principle for most 2 and 3-digit subtractions.

This means that once they have been introduced to the column method they have an alternative approach that is often preferable, depending upon the numbers involved.

The number line method also gives those children who can't remember or successfully apply the column method an approach that will work with any numbers (even 4-digit numbers and decimals) if needed.

It is advisable to spend at least the first term in Year 3 focusing upon the number line / counting up approach as a jotting through regular practice, while resources such as Base 10 are being used to explore decomposition practically. The column method can then be introduced in the 2nd / 3rd term once the understanding is secure.

Ideally, whenever columns are introduced, the expanded method should be practiced in depth (potentially up until 4-digit calculations are introduced). This should be done firstly with apparatus to build up a visual picture, and then gradually developed into the column procedure.

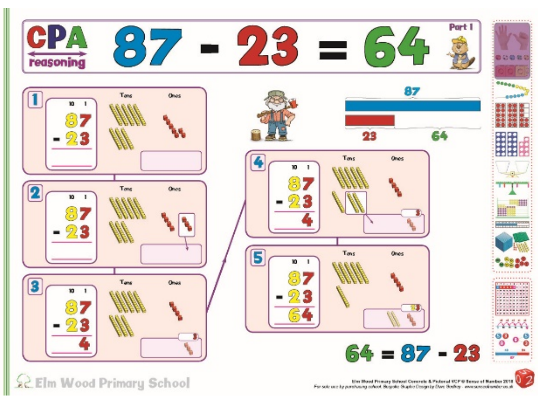
Y3/4

The expanded method of subtraction is an excellent way to introduce the column approach as it maintains the place value and is much easier to model practically with place value equipment such as Base 10 or place value counters.

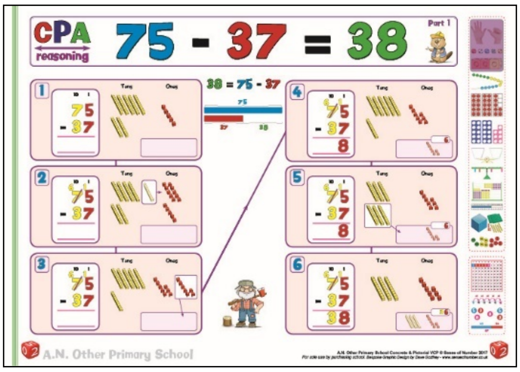
Introduce the expanded method with 2-digit numbers, but only to explain the process. Column methods are very rarely needed for 2-digit calculations.

Give the children ample opportunity to extend their place value skills into column subtraction by 'making' the calculation and explaining the process <u>before</u> writing it down.

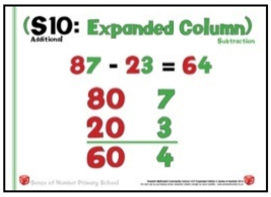
Partition both numbers into tens & ones, firstly with no exchange then exchanging from tens to ones.



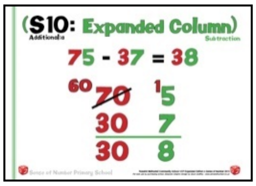
Make sure that children are explaining the process of exchanging / regrouping whilst using the materials. Give them many opportunities to take the 1 Ten and exchange / regroup as 10 Ones before writing this down in expanded form.



87 – 23



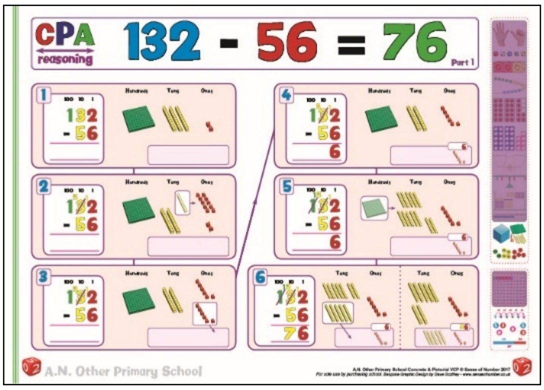
75 – 37



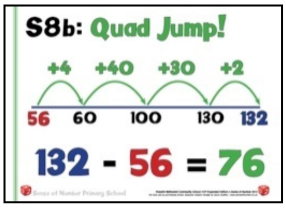
Develop into exchanging from hundreds to tens and tens to ones, always 'making' and discussing with apparatus first. Be explicit in explaining how the apparatus leads to the column procedure.

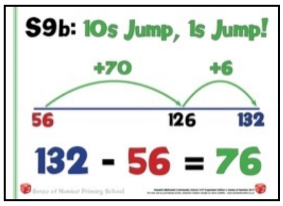
N.B. Please note the layout of the Base 10 resources in Tens Frame style, so that the groups of 5 can be seen immediately and don't need to be re-counted.

132 – 56

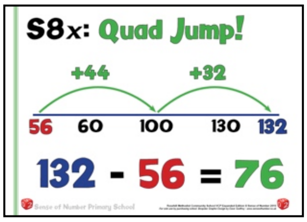


The number line method is equally as effective when crossing the hundreds boundary, either by the triple / quad jump method or by counting in tens then ones.

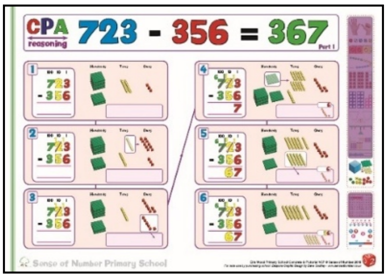


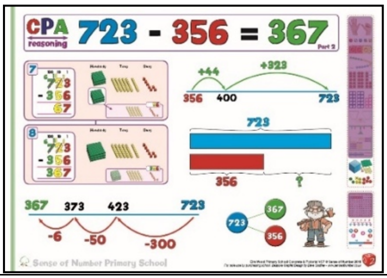


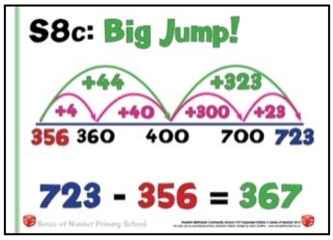
The 'quad jump' can be completed by many children in fewer steps, either a triple or **double** jump.

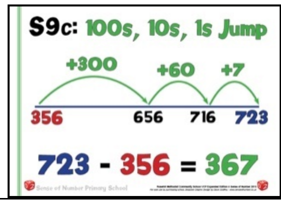


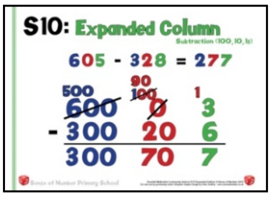
<table>
<tr>
<td>A</td>
<td>Move towards exchanging from hundreds to tens and tens to ones, in two stages if necessary. Use practical apparatus first at all times, especially when dealing with 3-digit calculations.



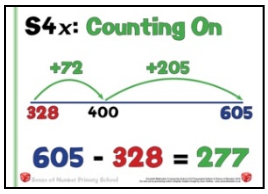

</td>
<td>The example below shows 2 alternatives, for children who need different levels of support from the image.


</td>
</tr>
<tr>
<td></td>
<td>For examples where exchanging is needed, then the number line method is equally as efficient, and is often easier to complete</td>
<td>As before, many children prefer to count in hundreds, then tens, then ones.


</td>
</tr>
<tr>
<td>B</td>
<td>Use some examples which include the use of zeros



Continue to use expanded subtraction until both number facts and place value are considered to be very secure!</td>
<td>For numbers containing zeros, counting up is often the most reliable method.


</td>
</tr>
</table>

Stage 3	Standard Column Method (decomposition)	
	Subtraction by counting back **Standard Method**	**Subtraction by counting up** **Number Lines (continued)**

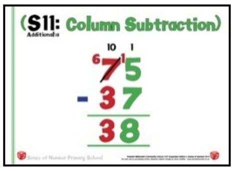
Mainly

Y4+

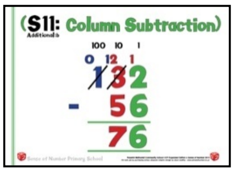
Decomposition relies on secure understanding of the expanded method, and simply displays the same numbers in a contracted form.

As with expanded method, and using practical resources such as place value counters to support the teaching, children in Years 3 or 4 (depending when the school introduces the column procedure) will quickly move from decomposition via 2-digit number 'starter' examples to 2 / 3 digit and then 3-digit columns.

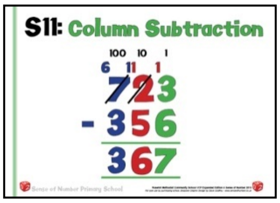
75 – 37



132 - 56



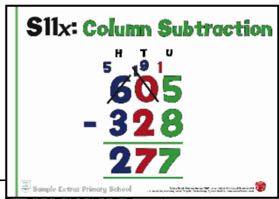
723 – 356



Continue to refer to digits by their actual value, not their digit value, when explaining a calculation. E.g. One hundred and twenty subtract fifty.

Again, use examples containing zeros, remembering that it may be easier to count on with these numbers (see Stage 2)

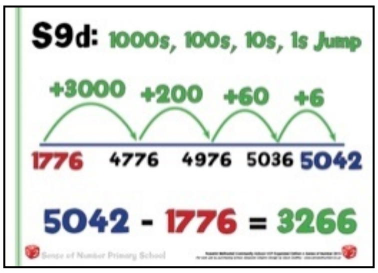
605 – 328

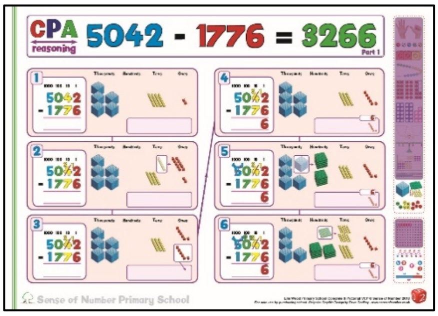


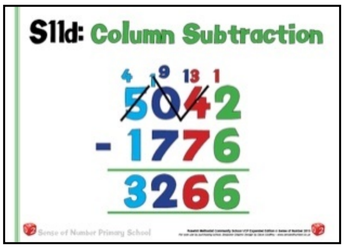
Y4

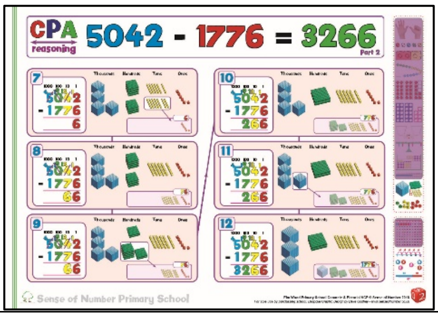
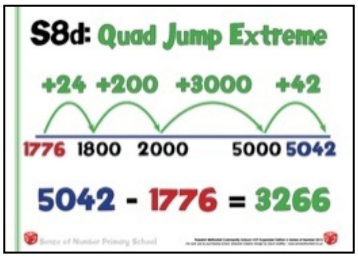
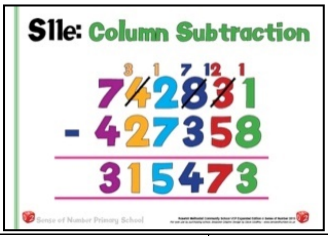
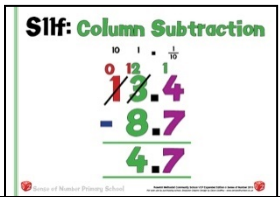
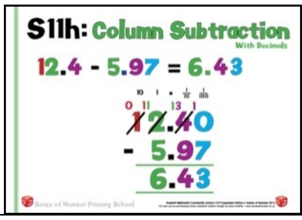
From late Y4 onwards, move onto examples using 4-digit (or larger) numbers and then onto decimal calculations.

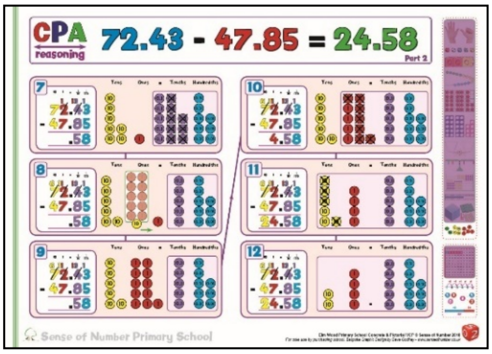
5042 – 1776

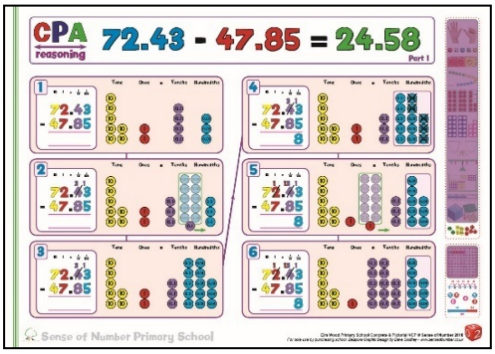


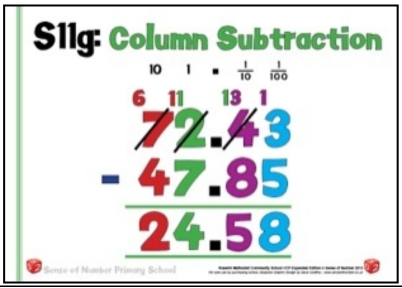
<table>
<tr>
<td></td>
<td>If necessary, apparatus can still be used to demonstrate the exchange / regroup principle





</td>
<td>
</td>
</tr>
<tr>
<td>Y5/6</td>
<td colspan="2">In Years 5 & 6 apply to any ‘big number’ examples.

</td>
</tr>
<tr>
<td></td>
<td>13.4 – 8.7

</td>
<td></td>
</tr>
<tr>
<td></td>
<td>12.4 – 5.97

</td>
<td></td>
</tr>
<tr>
<td></td>
<td>72.43 – 47.85</td>
<td></td>
</tr>
</table>

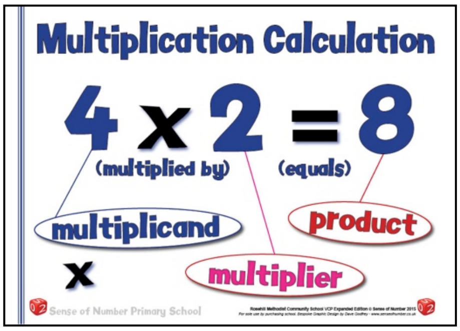
<table>
<tr>
<td></td>
<td>Even with calculations to 2 decimal places, practical apparatus can be used initially to explore / embed understanding or at a later stage for the children to demonstrate greater depth. In these instances, they can recreate a column method with decimals and explain each stage of the procedure.






</td>
<td></td>
</tr>
</table>

Multiplication Progression



The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to using an efficient method for

- two-digit by one-digit multiplication by the end of Year 3,
- three-digit by one-digit multiplication by the end of Year 4,
- four-digit by one-digit multiplication **and** two/three-digit by two-digit multiplication. by the end of Year 5
- three/four-digit by two-digit multiplication **and** multiplying 1-digit numbers with up to 2 decimal places by whole numbers by the end of Year 6.

To multiply successfully, children need to be able to:

- **recall all multiplication facts to 12 × 12;**
- **partition numbers into multiples of one hundred, ten and one;**
- **work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;**
- **similarly apply their knowledge to simple decimal multiplications such as 0.7 x 5, 0.7 x 0.5, 7 x 0.05, 0.7 x 50 using the related fact 7 × 5 and their knowledge of place value;**
- **add two or more single-digit numbers mentally;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **add combinations of whole numbers using the column method (see above).**

Note:

Children need to acquire **one efficient written method of calculation for multiplication, which** they know they can rely on **when mental methods are not appropriate.**

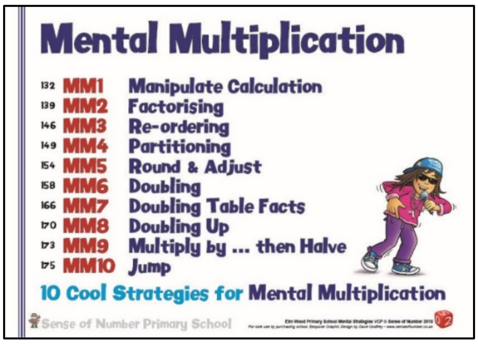
It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

Mental Multiplication Strategies

In a similar way to addition, multiplication has a range of mental strategies that need to be developed both before and then alongside written methods (both informal and formal).

Some of these are the same strategies used for addition but adapted for multiplication.

Others are specifically multiplication strategies, which enable more difficult calculations to be worked out much more efficiently.



Tables Facts

In Key Stage 2, however, before any written methods can be securely understood, children need to have a bank of multiplication tables facts at their disposal, which can be recalled instantly.

The learning of tables facts does begin with counting up in different steps, but by the end of Year 4 it is expected that most children can instantly recall all facts up to 12 x 12.

Doubles & Halves

The other facts that children need to know by heart are **doubles and halves.** As a general guidance, children should know the following doubles & halves: -

Year Group	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Doubles and Halves	All doubles and halves from double 1 to double 10 / half of 2 to half of 20	All doubles and halves from double 1 to double 20 / half of 2 to half of 40 (E.g.double 17=34, half of 28 = 14)	Doubles of all numbers to 100 with units' digits 5 or less, and corresponding halves (E.g. Double 43, double 72, half of 46) Reinforce doubles & halves of all multiples of 10 & 100 (E.g. double 800, half of 140)	Addition doubles of numbers 1 to 100 (E.g. 38 + 38, 76 + 76) and their corresponding halves Revise doubles of multiples of 10 and 100 and corresponding halves	Doubles and halves of decimals to 10 – 1 d.p. (E.g. double 3.4, half of 5.6)	Doubles and halves of decimals to 100 – 2 d.p. (E.g. double 18.45, half of 6.48)

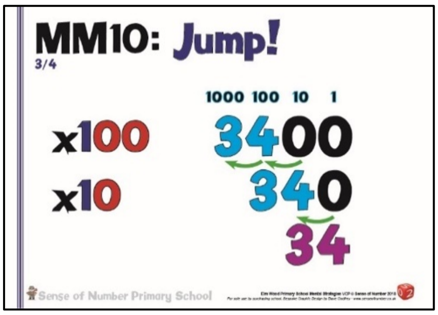
Once the children have learnt their doubles and their tables facts, there are then several mental calculation strategies that need to be taught so that children can continue to begin any calculation with the question ***'Can I do it in my head?'***

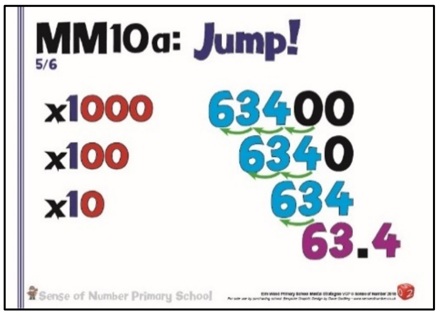
The majority of these strategies are usually taught in Years 4 – 6, but there is no reason why some of them cannot be taught earlier as part of the basic rules of mathematics.

Multiplying by 10 / 100 / 1000

This method is usually part of the Year 5 & 6 teaching programme for decimals, namely that digits move to the left when multiplying by 10, 100 or 1000, and to the right when dividing.

This also secures place value by emphasising that the decimal point doesn't ever move, and that the digits move around the decimal point





It would be equally beneficial to teach a simplified version of this method in KS1 / Lower KS2, encouraging children to move digits into a new column, rather than simply 'adding zeroes' when multiplying by 10/100.

The following 4 strategies for mental multiplication can be explicitly linked to 4 of the strategies in mental addition –

Partitioning, Round & Adjust, Re-Ordering (Number Bonds) & Manipulate the Calculation

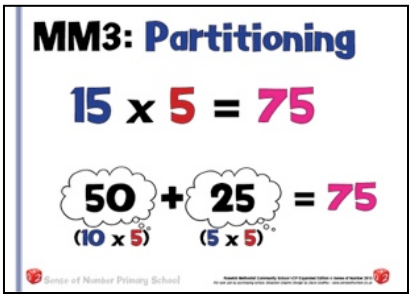
Partitioning

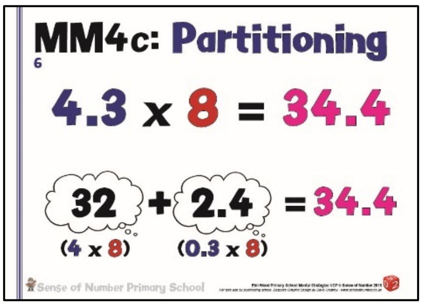
Partitioning is an equally valuable method for multiplication as it is for addition.

It can be quickly developed from a jotting to a method that is completed entirely mentally.

It is clearly linked to the grid method of multiplication, but should also be taught as a 'partition jot' so that children, by the end of Year 4, have become skilled in mentally partitioning 2 and 3 digit numbers when multiplying (with jottings when needed).

By the time children leave Year 6 they should be able to mentally partition most simple 2 & 3 digit, and also decimal multiplications, and should, wherever possible, be trying to work these out mentally.





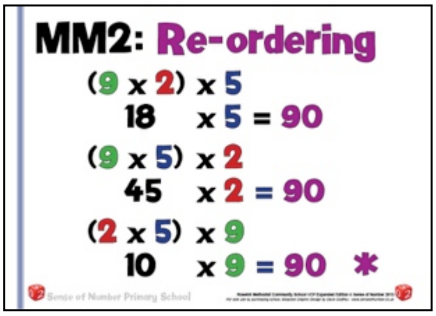
Round & Adjust

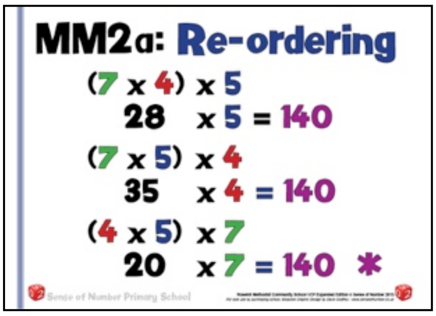
Round & Adjust is also a high quality mental method for multiplication, especially when dealing with money problems in upper KS2. Once children are totally secure with rounding and adjusting in addition, they can be shown how the method extends into multiplication, where they round then adust by the multiplier.

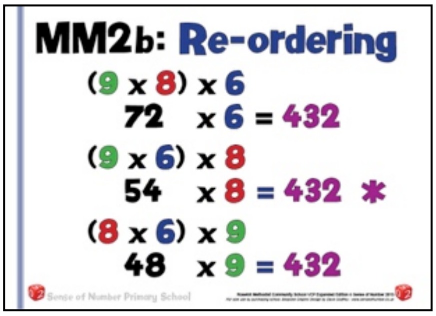
Re-ordering

Re-ordering is similar to **Number Bonds** in that the numbers are calculated in a different order. With Number Bonds in addition the children look for a simple bond that will make the numbers easier to add.
In Re-ordering, the children look at the numbers that need to be multiplied, and, using commutativity, rearrange them so that the calculation is easier to complete.

The slides below show various examples of re-ordering.
Each slide re-orders the calculation in 3 ways. The asterisked calculation in each of the examples is probably the easiest / most efficient rearrangement of the numbers. For example, when multiplying 7 x 4 x 5 it is much quicker to multiply the 5 x 4 first.







Manipulate Calculation

In addition, when applying this method, the children ‘passed over’ part of one number to the other in order to simplify the calculation.

In multiplication, however, they look at the numbers involved and determine whether an easier calculation can be created by dividing one of the numbers by a chosen amount and then multiplying the other by the same amount

This is probably the best method available for simplifying a calculation quickly, as long as the numbers being multiplied are appropriate.

Doubling strategies are probably the most crucial of the mental strategies for multiplication, as they can make difficult long multiplication calculations considerably simpler.

Doubling Tables Facts

Initially, children are taught to double one table to find another.
E.g. Doubling the 4s to get the 8s (E.g. 1 below)
If 4 x 6 = 24 then 8 x 6 must be 48 because 8 is double 4 (4 x 12 is also 48 as 12 is double 6)

This can then be applied to any table: -

If 8 x 7 = 56 then 16 x 7must be 112 because 16 is double 8 (or 8 x 14 = 112)

If 11 x 12 is 132 then 22 x 12 must be 264 because 22 is double 11 (or 11 x 24 = 264)

Doubling Up

This method develops the above method further, enabling multiples of **4, 8** and **16** onwards to be calculated by constant doubling.

It is linked very closely to the mental division 'halving' method, where we can divide by 8 by halving three times.

Multiplying by 10 / 100 / 1000 then halving / quartering

The final doubling / halving method works on the principle that multiplying by 10 / 100 is straightforward, and this can enable you to easily multiply by 5, 50 or 25.

Because 5 is half of 10 you can multiply a number by 10 then half it.

E.g. 86 x 10 = 860 so 86 x 5 must be 430.

In a similar way, 56 x 100 = 5600 so 56 x 25 must be a quarter of that amount (1400)

Factorising

The final mental method within the policy is factorising.

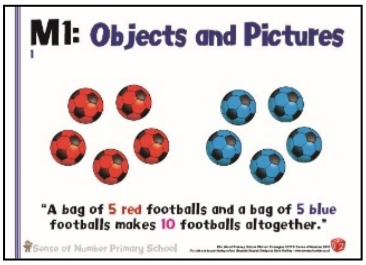
If children use their tables knowledge they can re-write a calculation to simplify it.

15 = 5 x 3 so multiplying 32 x 15 can be simplified as 32 x (5 x 3). 32 x 5 = 160 and 160 x 3 = 480

Multiplying a 2-digit number by 24, for example, may be easier if multiplying by a factor pair of 24. 52 x 24 = 52 x (4 x 6) 52 x 4 is a simple 208 then 208 x 6 is also a relatively straightforward 1248

This calculation could also be factorised as 52 x (8 x 3). 52 x 8 = 416. 416 x 3 = 1248

Written Methods of Multiplication

Stage 1	Number Lines, Arrays & Mental Methods
FS	In Early Years, children are introduced to grouping, and are given regular opportunities to put natural resources and real life objects into groups of 2, 3, 4, 5 and 10. They also stand in different sized groups, and use the term 'pairs' to represent groups of 2. This is then developed into using resources such as Base 10 apparatus, multi-link or an abacus. Children begin to visualise counting in ones, twos, fives and tens, saying the multiples as they count the pieces. E.g. Saying '10, 20, 30' or 'Ten, 2 tens, 3 tens' whilst counting Base 10 pieces
Y1	 Begin by introducing the concept of multiplication as repeated addition. Before using mathematical apparatus, use real objects and equipment such as cups, cakes, footballs, pencils, apples etc.) Children will firstly make then draw these objects in groups giving the product by counting up in 2s, 5s, 10s and beyond, and finally by writing the multiplication statement. The picture above will begin as an addition 'story' (**5 footballs and 5 footballs make 10 footballs**) but will then be written as a multiplication calculation (**5x 2 = 10**)

Make sure from the start (as explained in the introduction to this section) **that all children say the multiplication fact the correct way round, using the word ‘multiply’ more often than the word ‘times’ so that they understand what ‘multiplication’ means’**

For the example above, there are 5 footballs in 2 groups, showing **5 multiplied by 2 (5x2), not** 2 times 5. It is the **‘5’** which is being repeatedly added / **scaled up** / made bigger / **multiplied**.

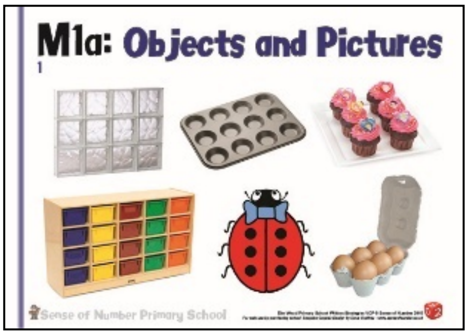
‘5 multiplied by 2’ shows ‘2 groups of 5’ or ‘Two fives’

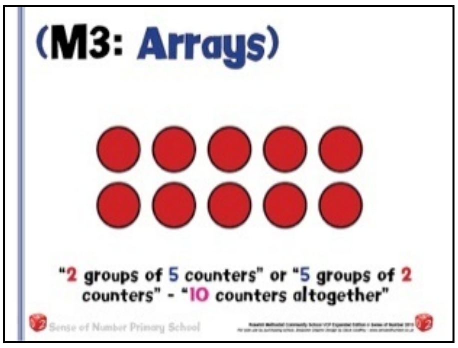
Again, using resources and real life objects, involve the children in telling stories and creating pictures for the 3s and 4s, and then into writing multiplication statements.

E.g. **3 footballs + 3 footballs + 3 footballs + 3 footballs would show 3 x 4 = 12 footballs**

Y2

The array





Build on children’s understanding that multiplication is repeated addition, using arrays and number lines to support their thinking.

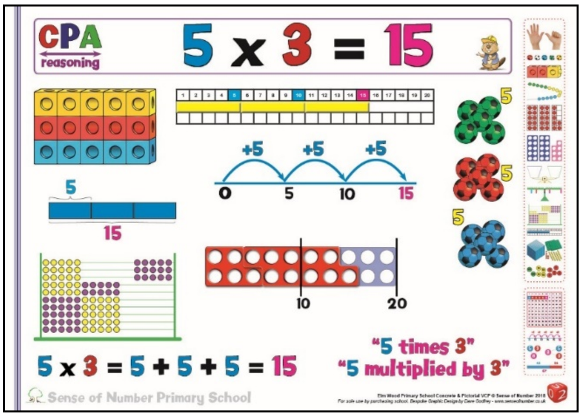
Start to develop the use of the array to show linked facts (commutativity). .

Make arrays with a wide range of objects, especially those which naturally occur in real-life such as windows, egg boxes, drawers or cake trays.

Emphasise that all multiplications can be worked out either way (2 x 5 = 5 x 2 = 10) as this will support the children in the future learning of their tables facts.

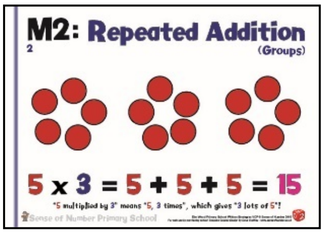
Practice counting in both steps (E.g. Both 2s and 5s) to prove that the answer is the same no matter which way round they are counted.

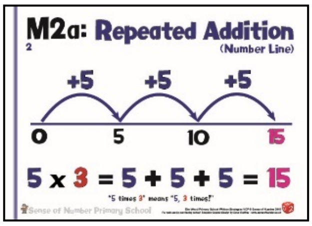
Encourage the children to see how the array makes it easier to see the calculation as a multiplication rather than a repeated addition.

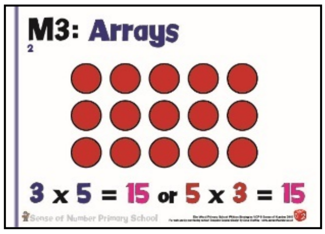


As with addition & subtraction, the key to developing clear understanding of multiplication is to involve the children in visualizing calculations with a wide range of resources. Using manipulatives such as, abacuses, number rods & multi-link, children can demonstrate & discuss their interpretation of multiplication statements

The above ‘materials can then be used alongside simple pictures and jottings which support the transition from repeated addition to multiplication.





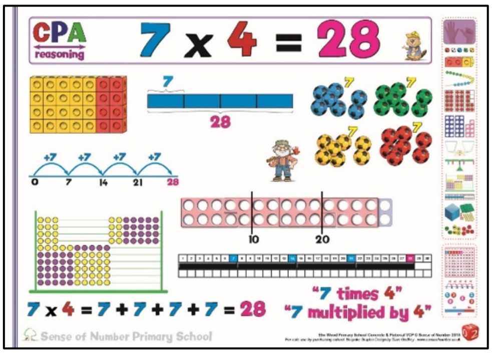


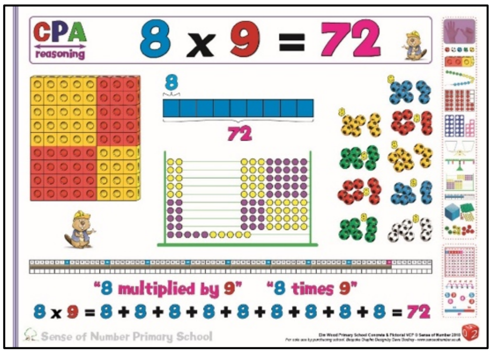
Y3

&

Y4

Extend the above to include the 3, 4, 8 and 11 times tables (Year 3)
then the 6, 7, 9 & 12 times tables (Year 4)



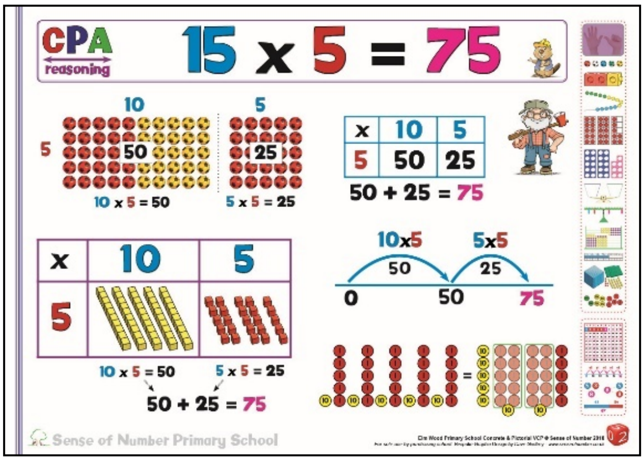


Even when dealing with larger tables such as the 7's and the 8's it is crucial that children can create a model or image of the calculation.
The footballs simply show four 7s and nine 8's as a real life picture, but the other images support children in either seeing the product or understanding the calculation in more depth.

The **Multi-link** and **Abacus** images are especially useful as they allow the **7 times table** to be seen as a combination of the **5s and 2s** and the **8 times table** as **5s and 3s.**
E.g. the 7 x 4 multi-link / abacus images are colour-coded to show a 5x4 array and a 2x4 array. The 8 x 9 images show 3x9 and 5x9.

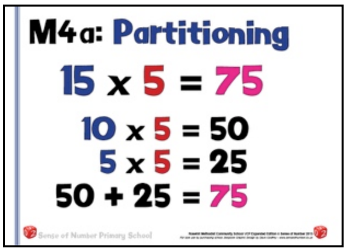
The Number Rod image places the four 7s / nine 8s on a track to display the products 28 / 72,
If the children become accustomed to using resources to support their counting, then use similar resources when displaying a visual for multiplication, they will automatically feel more confident when asked to picture / create / visualize a higher level calculation such as those later in this policy.

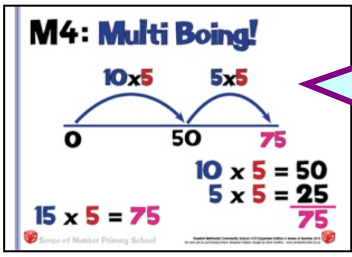
Extend the use of resources for 2 digit x 1 digit calculations so that children can visualize what the calculation looks like before they are taught any specific written methods or jottings.



Y3

Once the general models and images for multiplication are secure, begin to partition the 2 digit number using **jottings and number lines.**





Each of these methods can be used in the future if children find expanded or standard methods difficult.

Extend the methods above to calculations which give products greater than 100.

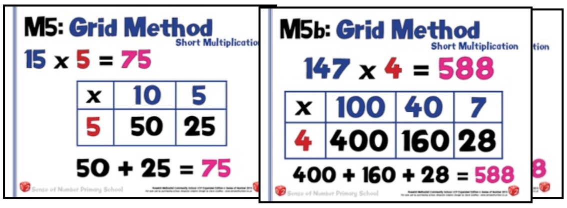
Use of 'Grid' Method within the New Curriculum

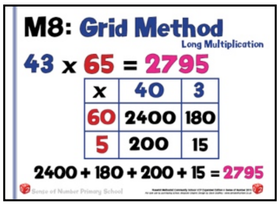
In the New Curriculum (2014), the Grid Method is <u>not</u> exemplified as a written method for multiplication. The only methods specifically mentioned are column procedures.

Most schools in the UK, however, have effectively built up the use of the grid method over the past 15 years, to the extent that it is generally accepted as one of the most appropriate 'written' methods for simple 2 and 3 digit x single digit calculations (**short multiplication**).

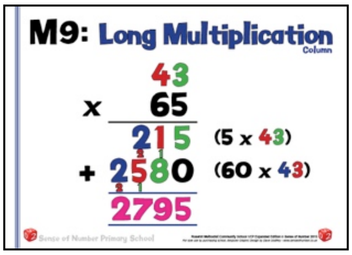
It develops clear understanding of place value through partitioning as well as being an efficient method, and is especially useful in Years 4 and 5.

Consequently, grid method is a key element of this policy, but, to align with the New Curriculum, is classed as a mental 'jotting'. It builds on partitioning, and is also the key mental multiplication method used by children in KS2

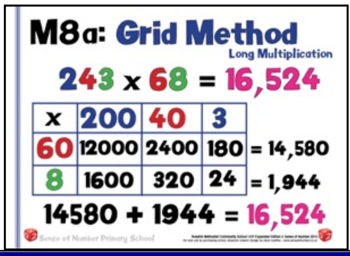


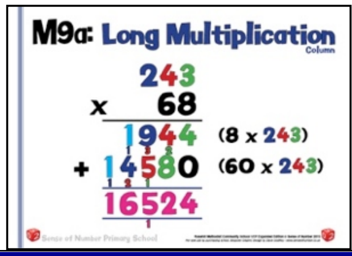


Column procedures still retain some element of place value, but, particularly for long multiplication, tend to rely on memorising a 'method', and can lead to many children making errors with the method (which order to multiply the digits, when to 'add the zero', dealing with the 'carry' digits' etc.) rather than the actual calculation.



Once the calculations become more unwieldy (4 digit x 1 digit or 3 / 4 digit x 2 digit) then grid method begins to lose its effectiveness, as there are too many zeroes and part products to deal with. At this stage column procedures are far easier, and, once learnt, can be applied much quicker. Grid methods can still be used by some pupils who find columns difficult to remember, and who regularly make errors, but children should be encouraged to move towards columns for more complex calculations





Stage 2 Written Methods - Short Multiplication

Grid Multiplication (Mental 'Jotting')

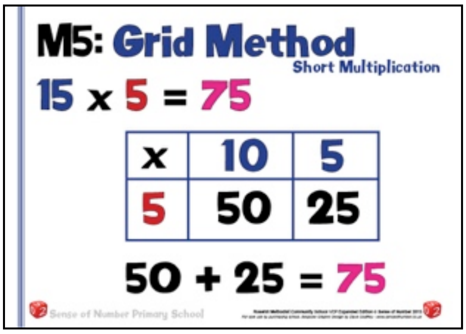
Column multiplication (Expanded method into standard)

Y3

Grid Multiplication

The **grid method** of multiplication is a simple, alternative way of recording the partitioning jottings shown previously.

As shown earlier, it can initially be taught using an array to show the actual product

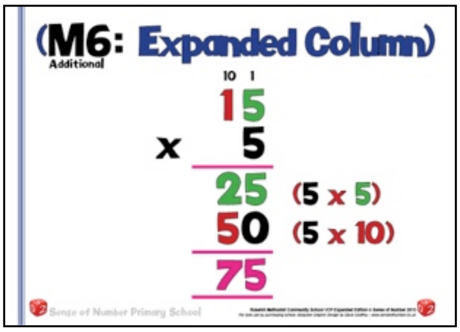


It is recommended that the grid method is used as the main method within Year 3. It clearly maintains place value, and helps children to visualise and understand the calculation better.

Column multiplication

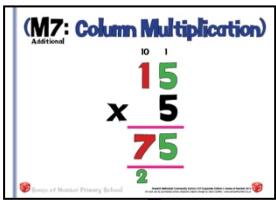
The expanded method links the grid method to the standard method.

It still relies on partitioning the tens and units, but sets out the products vertically.



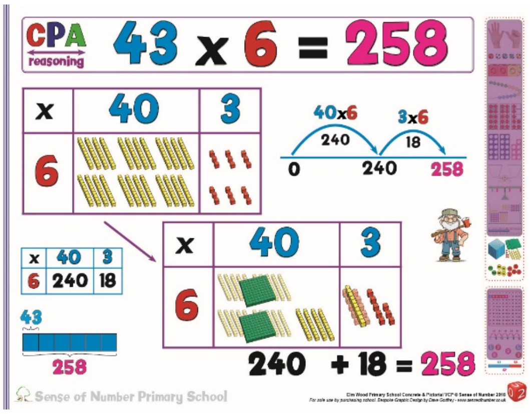
Children will use the expanded method until they can securely use and explain the standard method.

At some point within the year, the column method can be introduced, and children given the choice of using either grid or standard. Some schools may delay the introduction of column method until Year 4



Place the 'carry' digit below the line

When setting out calculations vertically, begin with the ones first (as with addition and subtraction).



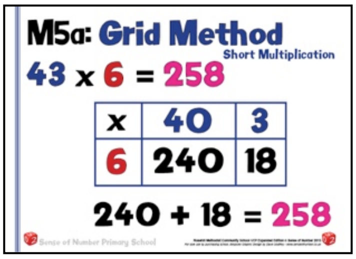
it is important to continue adopting the CPA approach by making and explaining the more complex 2 digit x 1 digit calculations.

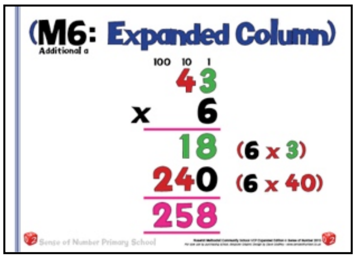
E.g. Using Base 10 apparatus, children can demonstrate 43 x 6 as 4 Tens repeated 6 times and 3 Ones repeated 6 times.

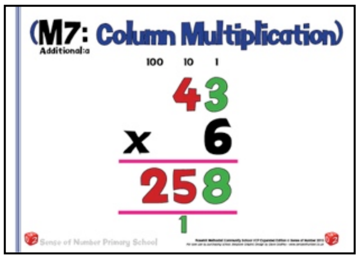
They can actually display this image within a Grid Method and then exchange / regroup 20 of the Tens into 2 Hundreds.

Continue to use apparatus regularly to support the place value and conceptual understanding of the calculation.

Y4

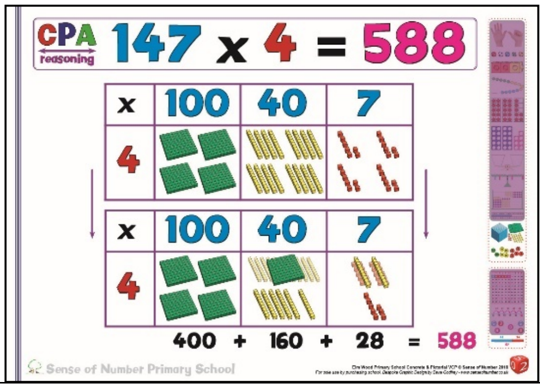






Continue to use **both grid** and **column methods** in Year 4 for more difficult **2 digit x 1 digit** calculations, extending the use of the grid method into mental partitioning for those children who can use the method this way and can simply jot down the sub products and answer.

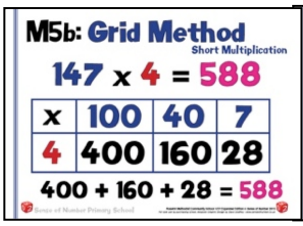
At this point, the expanded method can still be used when necessary (to help 'bridge' grid with column), but children should be encouraged to use their favoured method (grid or column) whenever possible.

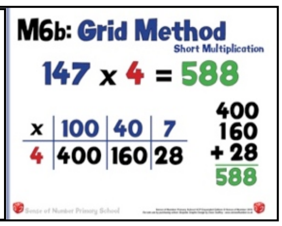


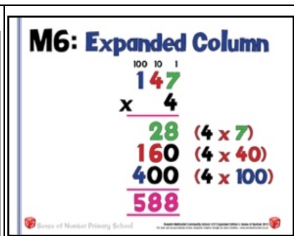
Using **apparatus** for **3 digit x 1 digit** calculations is an excellent way to continue developing the conceptual understanding of what the calculations look like and the actual size of the number that is being manipulated.

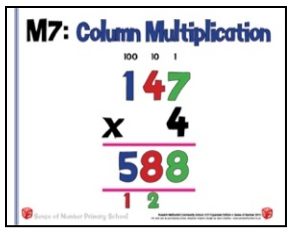
In the example displayed, children can create **100 x 4, 40 x 4 and 7 x 4** using Base 10 then show how the **16 Tens** can be exchanged / regrouped into **1 Hundred and 6 Tens**.

For 3 digit x 1 digit calcualtions, both grid and standard methods are efficient.
Continue to use the grid method to aid place value and mental arithmetic.
Use column method for speed, and to make the transition to long multiplication easier.
If both methods are taught consistently then children in Year 4 will have a clear choice of 2 secure methods, and will be able to develop both accuracy and speed in multiplication.



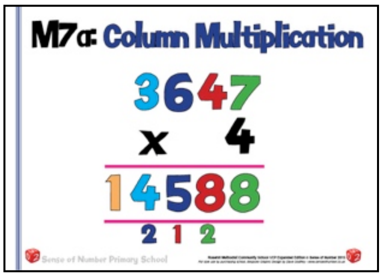


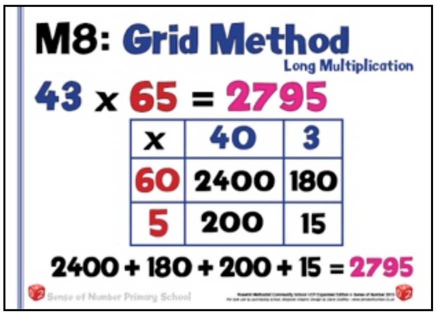
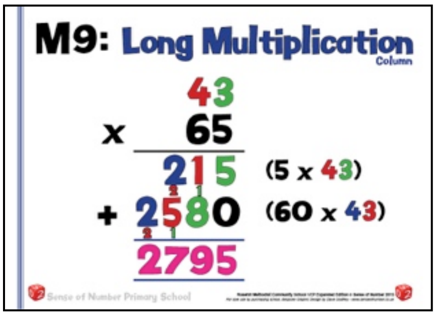


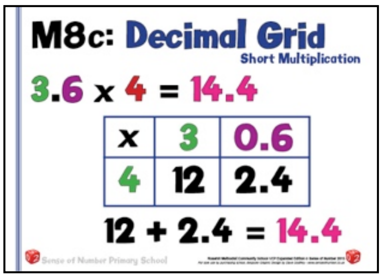
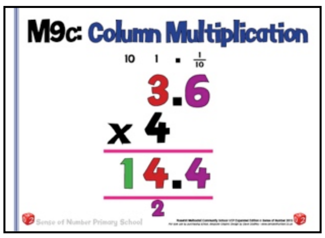


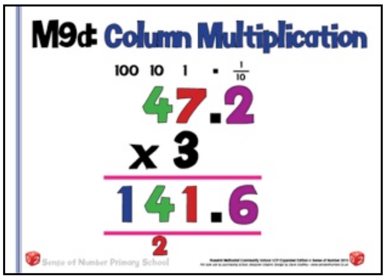
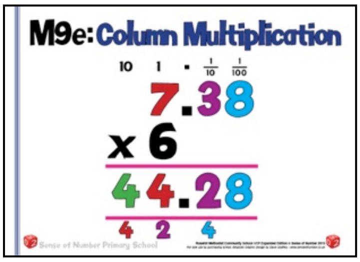
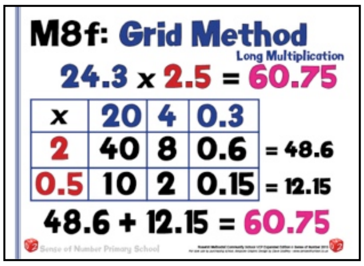
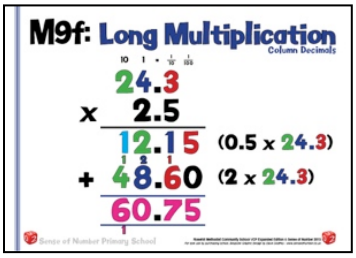
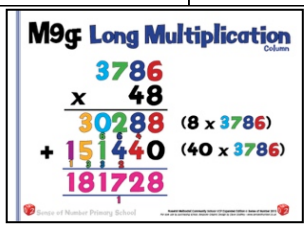
Sometimes children find the multiplication and place value parts of Grid Method to be fairly

Expanded method can still be used for children who need extra support with place value (and

	simple, but then struggle with the actual addition at the end (see 1st example above). In these instances, encourage them to complete the addition using a column method (see 2nd example above)	as a ‘bridging’ method between grid and column procedures. Once this is secure then they can practice the speed and security of the column method, but ensuring they can still explain the place value (using apparatus if necessary) when required
Y5	For a 4 digit x 1 digit calculation, the column method, once mastered, is quicker and less prone to error. The grid method may continue to be the main method used by children who find it difficult to remember the column procedure, or children who need the visual link to place value.	

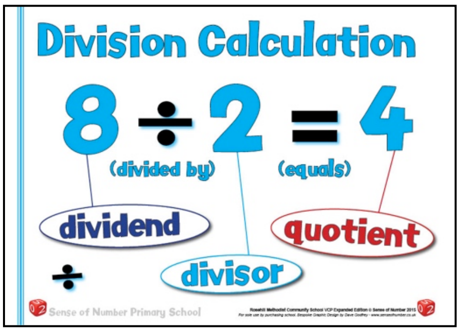
Stage 3	**Long Multiplication (TU x TU)**	
	Grid Multiplication	**Column multiplication** (Expanded method into standard)
Y5	Extend the grid method to TU × TU, asking children to estimate first so that they have a general idea of the answer. ***(43 × 65 is approximately 40 × 70 = 2800.)***  As mentioned earlier, grid method is often the ‘choice’ of many children in Years 5 and 6, due to its ease in both procedure and understanding / place value and is the method that they will mainly use for simple long multiplication calculations.	**Children should only use ‘standard’ column method for long multiplication if they can regularly get it correct using this method.**  There is no ‘rule’ regarding the position of the ‘carry’digits. Each choice has advantages and complications. Either carry the digits mentally or have your own favoured position for these digits.

	 Many children will find the use of Grid method as an efficient method for multiplying decimals. They must also practice mental partitioning for decimal calculations such as the one above.	 Extend the use of the column method into decimal multiplication. It is advisable to set out the columns as shown on the example above so that the place value remains secure.

<table>
<tr><td></td><td></td><td>It is also helpful to the children in showing how many digits will need to be displayed after the decimal point.</td></tr>
<tr><td>Y6</td><td></td><td>
</td></tr>
<tr><td></td><td></td><td>
</td></tr>
<tr><td></td><td></td><td>In the examples above, continue to think carefully about the layout of the calculation, keeping the place value accurate when multiplying.</td></tr>
<tr><td></td><td>
</td><td>
</td></tr>
<tr><td></td><td colspan="2">

By the time children meet 4 digits by 2 digits, the only efficient method is the standard method for Long Multiplication.</td></tr>
</table>

Division Progression



The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to long division through Years 3 to 6 – first using short division 2 digits ÷ 1 digit, extending to 3 / 4 digits ÷ 1 digit, then long division 4 / 5 digits ÷ 2 digits.

To divide successfully in their heads, children need to be able to:

- **understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;**
- **recall multiplication and division facts to 12 × 12, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;**
- **know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;**
- **understand and use multiplication and division as inverse operations.**

Children need to acquire **one efficient written method of calculation for division**, which they know they can rely on **when mental methods are not appropriate.**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

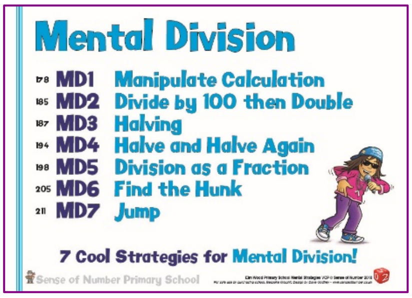
To carry out written methods of division successfully, children also need to be able to:

- **visualise how to calculate the quotient by visualising repeated addition;**
- **estimate how many times one number divides into another – for example, approximately how many sixes there are in 99, or how many 23s there are in 100;**
- **multiply a two-digit number by a single-digit number mentally;**
- **understand and use the relationship between single digit multiplication, and multiplying by a multiple of 10.**
  (e.g. 4 x 7 = 28 so 4 x 70 = 280 or 40 x 7 = 280 or 4 x 700 = 2800.)

Mental Division Strategies

In general, when faced with a division calculation, most people choose to use a written method. This is usually the formal 'bus stop' method of division, and tends to be the chosen method no matter which numbers are being divided.

There are, however, quite an extensive range of mental division strategies which can be used depending on the numbers involved. Children need to be encouraged to use their sense of number and to approach each calculation in the same way as addition, subtraction and multiplication, asking themselves ***'Can I do it in my head?'***



Doubles & Halves

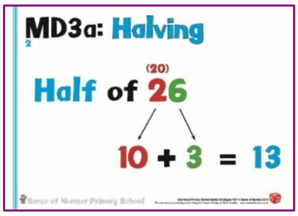
As mentioned earlier within the section on multiplication, children need to know by heart certain **doubles and halves.** As these are no longer mentioned explicitly within the National Curriculum, it is crucial that they are part of a school's mental calculation policy.

In a very similar way to doubling, if children haven't learnt to recall simple halves instantly, or, more importantly, haven't been taught strategies for mental halving, then they cannot access some of the mental calculation strategies for division (E.g. Halve twice to divide by 4, halve three times to divide by 8, divide by three then halve to divide by 6)

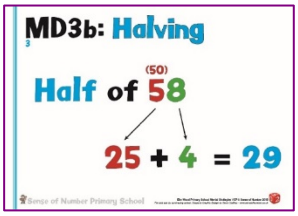
Halving (MD3)

Before certain doubles / halves can be recalled, children can use a simple partition jotting to help them record their steps towards working out a double / half. This begins with a straightforward halving of the Tens and halving of the Ones (see Y2 & Y3 examples below).

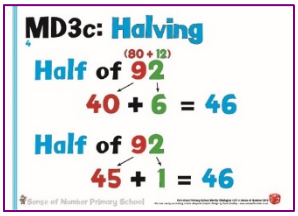
Y2



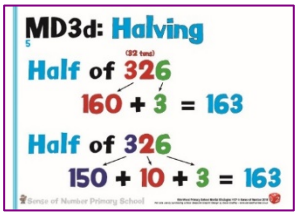
Y3



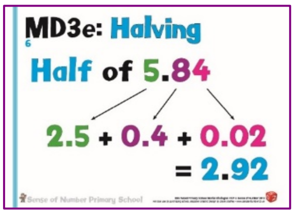
Y3/4



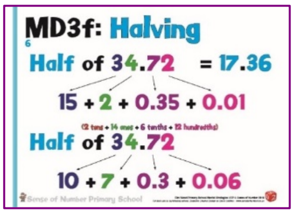
Y4/5



Y5



Y6



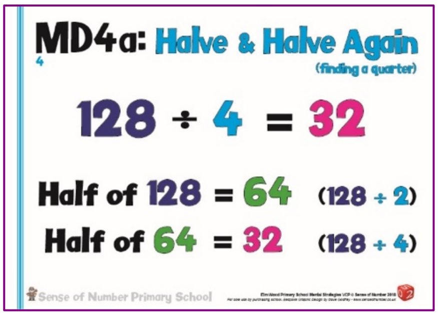
Once the children have learnt their halves (and their tables facts), there are then several mental calculation strategies that need to be taught so that children can continue to begin any calculation with the question ***'Can I do it in my head?'*** The majority of these strategies are usually taught in Years 4 – 6, but there is no reason why some of them cannot be taught earlier as part of the basic rules of mathematics.

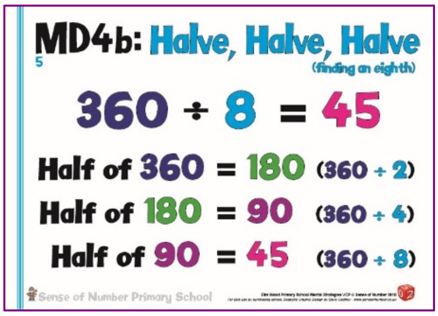
Halve & Halve (& Halve) Again (MD4)

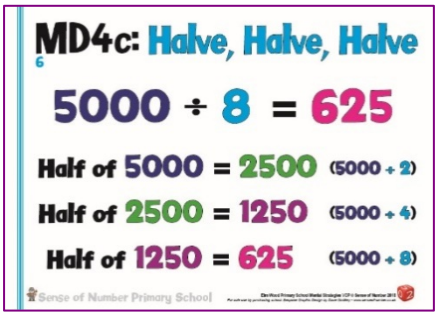
As outlined above, the main use of halving as a mental method is in order to make dividing by 4 or dividing by 8 much easier.
To divide 128 by 4, rather than using a written procedure, it makes more sense to simply halve the number twice.

Dividing 360 by 8 can be done several ways, including mentally chunking the number into 320 and 40 then dividing both parts by 8 (see 'Find the Hunk' method below). The easiest method, however, for many children (& adults!) is to halve the 360 three times.

Similarly, dividing 5000 by 8 would normally involve a bus stop method. This can be made far easier by halving 5000 to give 2500, halving again to give 1250 then halving a third time to leave a quotient of 625.

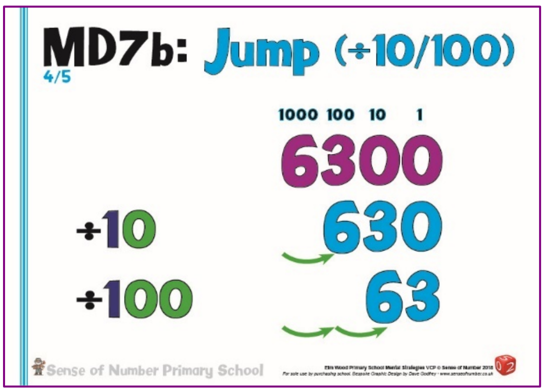


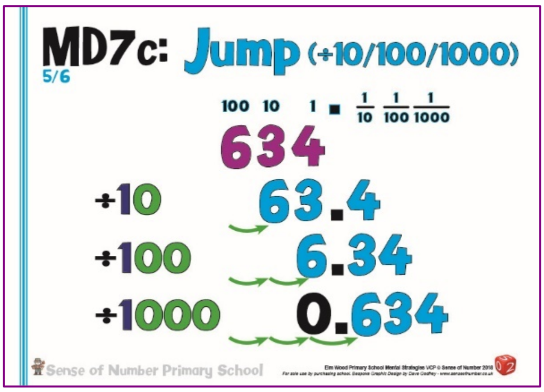




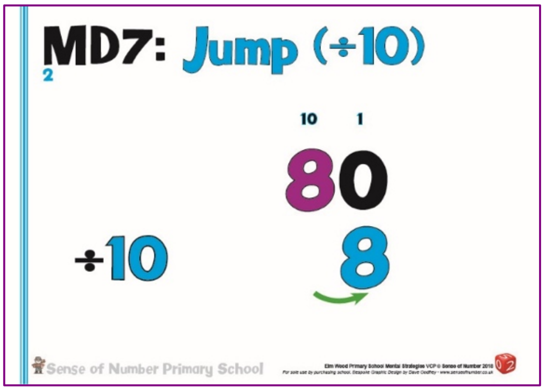
Dividing by 10 / 100 / 1000 – Jump! (MD7)

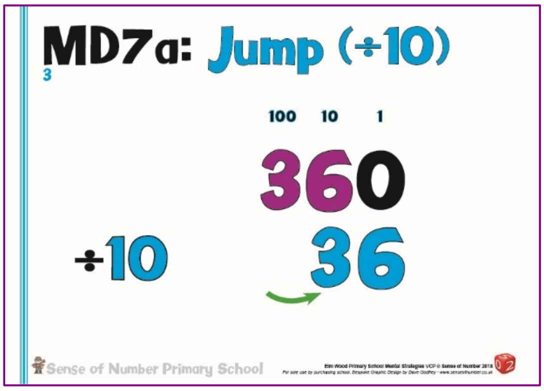
As with multiplication, this method is usually part of the Year 5 & 6 teaching programme for decimals, namely that digits **move to the left** when multiplying by 10, 100 or 1000, **and to the right** when dividing by 10 / 100 / 1000
This also secures place value by emphasising that the decimal point doesn't ever move, and that the digits move around the decimal point (not the other way around, as so many adults were taught at school).





It would be equally beneficial to teach a simplified version of this method in KS1 / Lower KS2, encouraging children to move digits into a new column, rather than simply 'removing zeroes' when dividing by 10/100.





Division as a Fraction (MD5)

This a much higher level method that is usually left until Year 6, or even at high school, but there is no reason why children cannot be introduced to it much earlier if they are secure in their understanding that every division can be written as a fraction and every fraction can be written as a division.

The earliest understanding of this method begins in Year 1, when children first meet the fractions ½ and ¼.
In almost all instances, they are given a simple picture of a half as '1 out of 2' and a quarter as '1 out of 4'. They are also introduced to the written fractions ½ and ¼.

The fraction line/bar (or the '***vinculum'***) is then described as meaning
'out of' (1 piece out of 4 equal pieces),
'shared between' (1 cake shared between 4 people)
or '**divided by'** (1 whole **divided by** / into / between 4)
These definitions continue in Key Stage 2 with other unit fractions.
1/6 is 1 out of 6,
1 cake shared between 6
1 whole **divided by** 6 / into 6 pieces / between 6

Unfortunately, it is rare for the actual division statement to also be written alongside the fraction.

½ could also be written as **1 ÷ 2.**
¼ could also be written as **1 ÷ 4.**
1/6 could also be written **as 1 ÷ 6.**

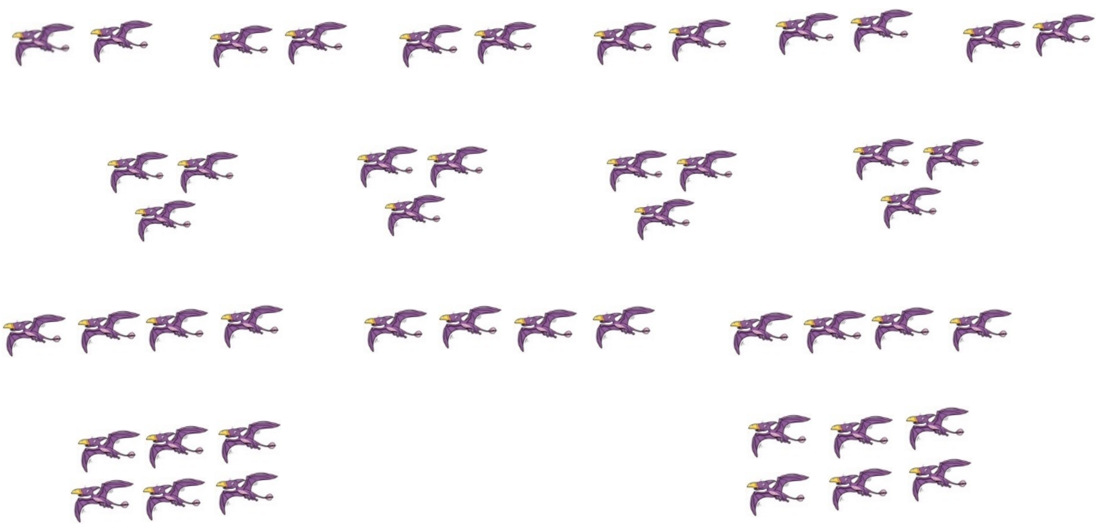
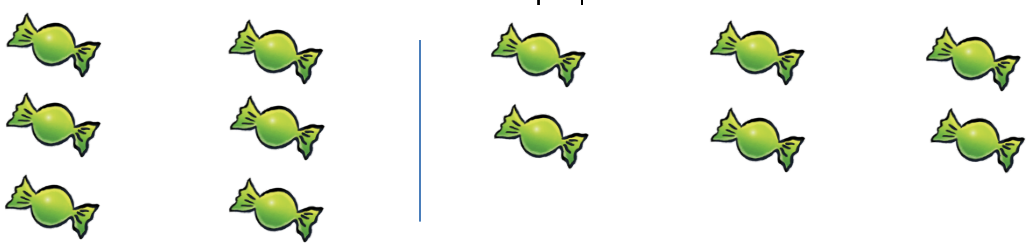
If this understanding was developed throughout the school then all fractions can be written and displayed in different ways.

The usual way to show 3/5 would be 3 out of 5 (as seen in the first picture below) – this picture is in effect showing 3/5 of 1 whole.

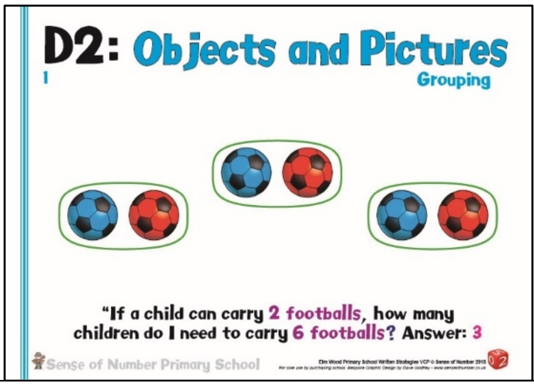
Therefore, children could be taught that every 'fraction' statement could potentially be made easier if written as a division statement.

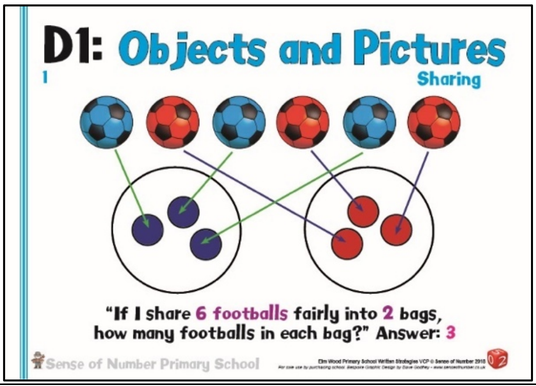
At a very simple level, ¼ **of 20** could be re-written (and displayed) as **20 ÷ 4**
or 1/8 of 24 could be re-written (and displayed) as **24 ÷ 8.**
Developing this further, a calculation that appears to be quite complex for a child within Year 4, such as ¼ of 3, can be answered extremely quickly if rewritten as a division and then manipulated into a fraction.
This method can then be applied alongside the understanding / method of converting improper fractions to mixed numbers in Years 5 & 6.
Consequently, calculations which appear to be extremely difficult to answer mentally can be rewritten as a mental jotting and then worked out quickly.

Written Methods Of Division

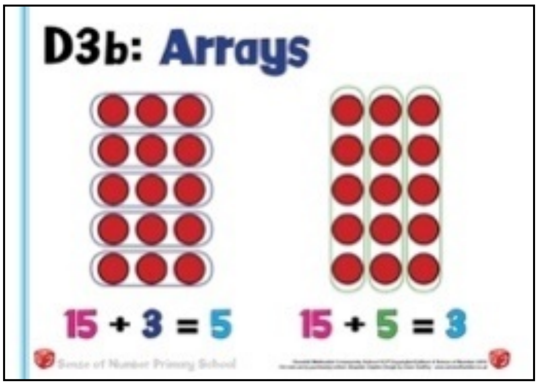
<table>
<tr><th rowspan="2">Stage 1</th><th colspan="2">Concepts and Number Lines (pre-chunking)</th></tr>
<tr><th>Grouping</th><th>Sharing</th></tr>
<tr><td>EYFS</td><td colspan="2">From EYFS onwards, children need to explore practically both grouping and sharing.
Links can then be made in both KS1 and KS2 between sharing and fractions.
For example, children can be asked to put their 12 dinosaurs into groups of 2, 3. 4 or 6.

The children could share 6 sweets between 2 or 3 people.
</td></tr>
</table>

Y1	Begin by giving children opportunities to use concrete objects, pictorial representations and arrays with the support of the teacher. Use the words ‘sharing’ and ‘grouping’ to identify the concepts involved. Before using mathematical apparatus, use real objects and equipment such as cups, cakes, footballs, pencils, apples etc. Children will firstly make then draw these objects by grouping and sharing them, working out the quotient by either counting the number of groups or the number within each group, then finally by writing the division statement.
	Make sure that the children have experience in representing the same division calculation in both ways for a range of different calculations. For example 6 ÷ 2 needs to be ‘made’ as both 6 in groups of 2 (How many 2s in 6?) and 6 shared between 2.





Y2	Identify the link between multiplication and division using the array image. Begin to build on children’s understanding that division is the inverse of multiplication, using arrays and number lines to support this thinking. Start to develop the use of the array to show that the same picture can be used to show both multiplication and division, and that the array demonstrates both sharing and grouping with one individual image.

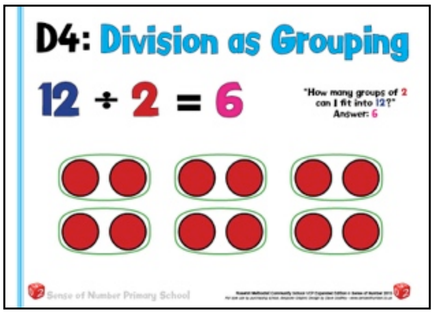


As with multiplication, make arrays using a wide range of objects, especially those which naturally occur in real-life such as windows, egg boxes, drawers or cake trays.

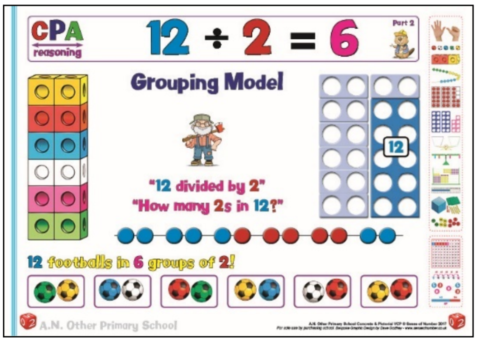
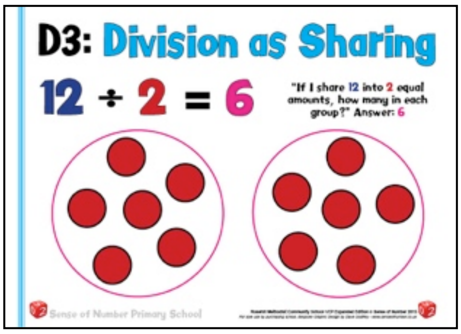
Emphasise that all simple divisions can be worked out and pictured both ways (sharing or grouping)

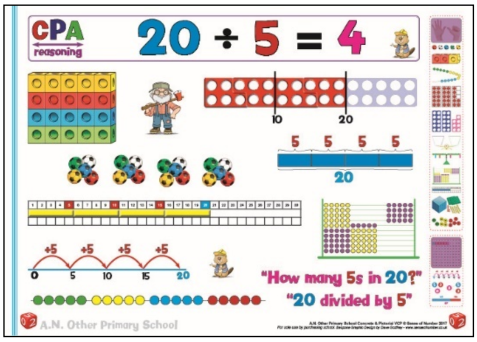
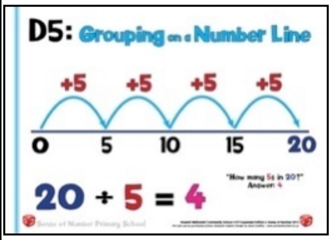
Encourage the children to see how the array makes it very easy to see both models of division, and practice circling the array depending on whether they want to show either: -

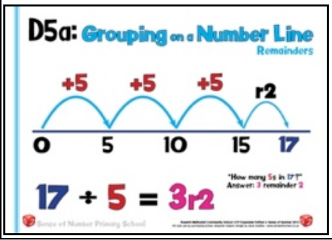
- Grouping demonstrated both ways (see above)
- Sharing demonstrated both ways (same pictures but opposite calculation written below each image)
- The same calculation as either sharing or grouping (the second image in the first picture above as 15 ÷ 5 (grouping) or 15 ÷ 3 (sharing)

<table>
<tr><td></td><td colspan="2"></td></tr>
<tr><td></td><td>Identify Grouping as the key model for simple division. Relate Grouping to knowledge of multiplication facts / the inverse of multiplication.


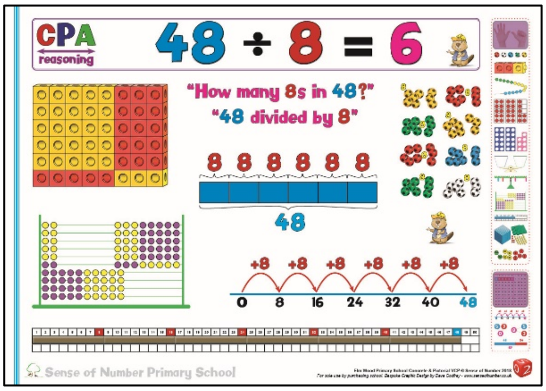
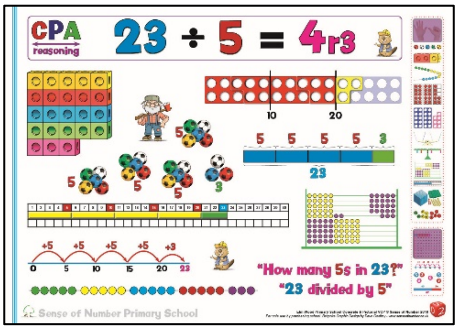
Regularly use the key vocabulary: 12 ÷ 2 means how many 2's can I fit into 12?'
or '20 ÷ 5 means how many 5's can I fit into 20?'

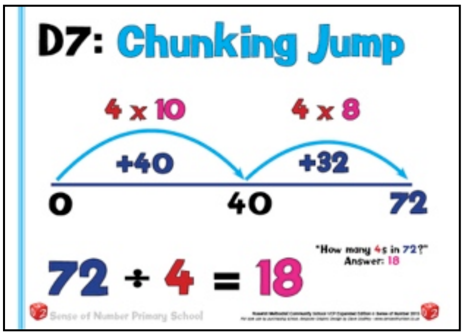
Begin to move away from real life objects / natural resources, instead using mathematical apparatus to show the same division calculation

</td><td>Identify Sharing as the secondary model of division.
Relate sharing to fractions, showing that sharing between 2 is the same as halving.

</td></tr>
</table>

<table>
<tr><td></td><td>
</td><td></td></tr>
<tr><td></td><td>Moving towards a slightly more abstract visual, but one which still maintains clear understanding, the number line is an excellent way to quickly show how counting on (rather than counting back) is the easiest way to use a number line when solving a division calculation.
This is particularly beneficial if dealing with remainders. Using the example below, it is far easier to count forwards in 5s to 15 then count on 2 than starting at 17 and counting backwards in 5s.



</td><td>Regularly stress the link between multiplication and division, and how children can use their tables facts to divide by counting forwards in steps.</td></tr>
</table>

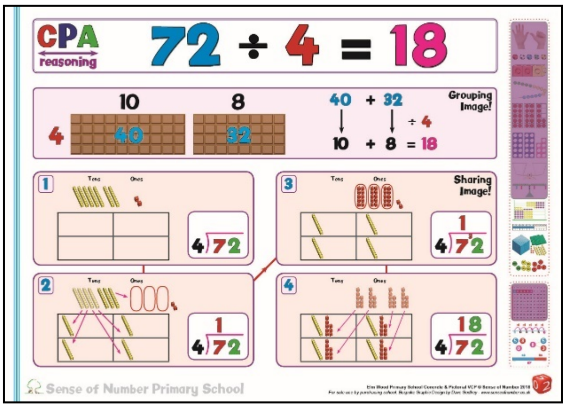
<table>
<tr><td>Y3</td><td>It is really important to maintain the use of concrete materials as the calculations become gradually more complex, and tables facts harder to instantly recall.

Children using apparatus such as number rods, abacuses, and cubes can still explain the division and have a better understanding of the calculation if they are asked to make it.

</td><td>Continue to give children practical images for division by grouping when dealing with simple remainders.
Asking the question 'How many 5s are there in 23?' allows the opportunity to visualize the remainder.

</td></tr>
</table>

<table>
<tr><th rowspan="2">Stage 2</th><th colspan="2">Chunking & Standard Methods</th></tr>
<tr><th>Standard Methods</th><th>Chunking
Find the Hunk &
NNS Chunking</th></tr>
<tr><td>Y3</td><td></td><td>

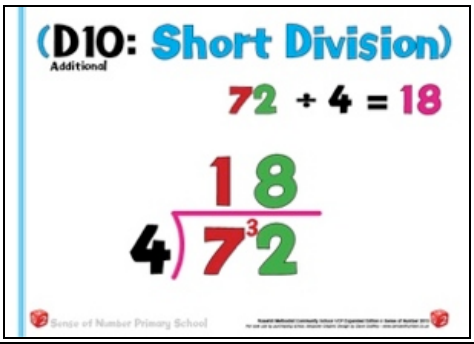
previously encountered in Y2, developing an understanding of division with the number line is an excellent way of linking division to multiplication. It can show division both as repeated subtraction, but it is simpler to show division by counting forward to find how many times one number 'goes into' another.</td></tr>
<tr><td></td><td colspan="2">These slides introduce the Short Division
(Bus Stop) method in Year 3.</td></tr>
</table>

It is recommended that when children start to use this method, it is only introduced once tables facts are relatively secure.



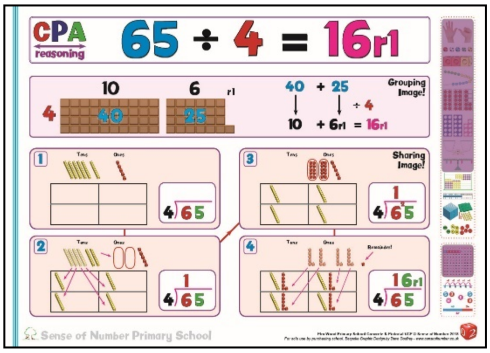
When introducing Short Division formally, use Base 10 and make sure you introduce it using the **sharing model (**as mentioned in the 'Teaching Short Division' explanation on the previous two pages)

The calculation starts with, 'I have 7 Tens, to share between 4. That's 1 Ten each with 3 remaining. These 3 Tens are regrouped into 30 Ones. The 32 Ones are now shared between 4– that's 8 Ones each.

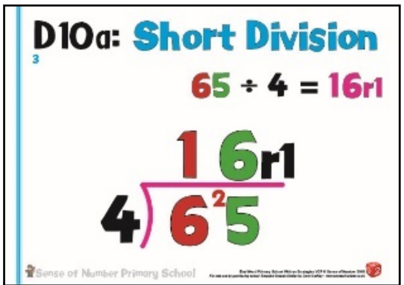


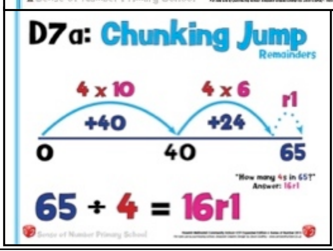
As mentioned previously, children should be taken to the standardised 'bus stop' format for short division only when they have mastered the use of apparatus and are able to explain the process.

At any stage they can revert back to the use of concrete materials to recap / aid their understanding, or to deal with higher level calculations in later year groups



Base 10 will still be used for the first principles of Concrete – Pictorial – Abstract before the children progress to the abstract method.





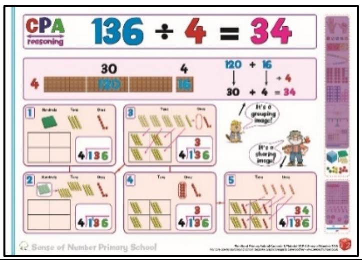
The number line provides an excellent image to show division with remainders, and can be used to support or instead of 'Find The Hunk'

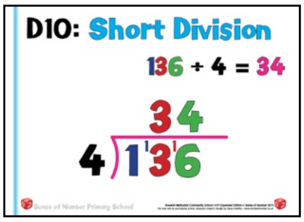
Y4

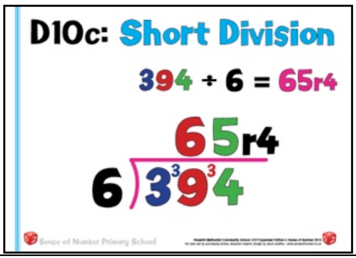
As described in more detail earlier, short division can now be developed into 3 digit dividends. Base 10 (or place value counters for children who have better understanding) is used tomodel and explain how the method works.

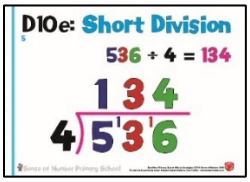
<table>
<tr><td></td><td>
The calculation starts with, ‘I have 1 Hundred, which cannot be shared (in Hundreds) between 4. The Hundred is regrouped into 10 Tens. The 13 Tens are now shared between 4 – 3 each. The remaining Ten is regrouped into 10 Ones. The 16 Ones are now shared between 4 – that’s 4 Ones each.

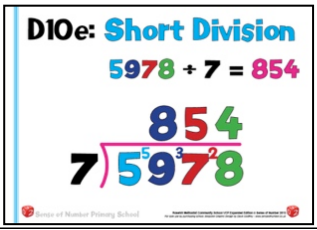
Once understood and explained with a range of calculations, children use the ‘bus stop’ method.





</td></tr>
<tr><td>Y5</td><td>


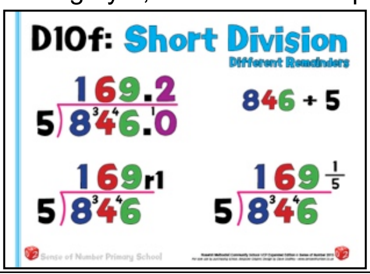
</td></tr>
<tr><td></td><td>


Once a child is confident with the apparatus then, as before, they can use the ‘bus stop’ method, explaining it when necessary.
</td></tr>
<tr><td></td><td>


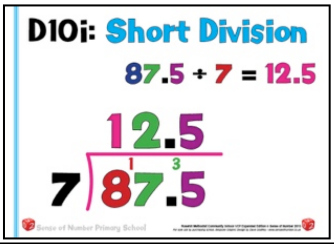
</td></tr>
<tr><td>Y6</td><td>
Once a child is in Upper Key Stage 2 they should be aware that giving a remainder as a whole number is not accurate enough.

A remainder of 1, for example, has a completely different meaning when the divisor changes. When dividing by 2 it represents ½ (or 0.5), when dividing by 4 it is ¼ (or 0.25), when dividing by 10 it is worth 1/10 (or 0.1).

In the example above, when dividing by 5, the remainder represents / is worth 1/5 (or 0.2).



</td></tr>
</table>

	D10i: Short Division 87.5 ÷ 7 = 12.5 12.5 7) 87.5
	D12: Long Division Short Division Method 26r21 37) 983